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HYDRODYNAMIC CONCEPTS SELECTED
TOPICS FOR UNDERWATER NUCLEAR
EXPLOSIONS

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HYDRODYNAMIC CONCEPTS
SELECTED TOPICS FOR UNDERWATER NUCLEAR EXPLOSIONS

by
Hans G. Snay

ABSTRACT: This paper gives a narrative description of the hydrodynamic concepts which are important for the understanding of underwater explosion processes with particular emphasis given to the physical background. Mathematical developments are entirely omitted or kept to a minimum.

The paper describes the concepts of the various fluid motions and fluid models involved in explosion phenomena. The properties of the shock front are described and the interrelationship between the formation of the nuclear bubble and the shock front is pointed out. The properties of high amplitude waves are outlined using Riemann's description. The formation of underwater explosion bubbles is shown to be a hydrodynamic consequence of the spherical pressure wave emitted by the explosion. Such a wave always produces a radial mass flow directed outward, the afterflow, which must lead to the formation of a cavity. The acoustic approximation of pressure waves is discussed in Appendix A. Appendix B contains comments on the entropy concept.

EXPLOSIONS RESEARCH DEPARTMENT

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND
20910

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HYDRODYNAMIC CONCEPTS - SELECTED TOPICS FOR UNDERWATER
NUCLEAR EXPLOSIONS

This report was written for inclusion as Chapter II in the book,
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and suggestions as to the final version will be highly appreciated.
They should be sent to Commander, U. S. Naval Ordnance Laboratory,
Attention: Dr. H. G. Snay, White Oak, Maryland.

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<div style="text-align:right">

A. A. BARTHES
Captain, USN
Acting Commander

C. J. ARONSON
By direction

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HYDRODYNAMIC CONCEPTS
SELECTED TOPICS FOR UNDERWATER NUCLEAR EXPLOSIONS

I. INTRODUCTION

1.1 <u>OBJECTIVE</u>. It is the objective of this paper to lay down the funda-
mental concepts that are needed to fully appreciate the effects of underwater
nuclear explosions. Clearly, it is the aim of any presentation on this subject
(a) to explain the experimental results of underwater nuclear explosion tests
and (b) to interpret these so that the best possible and most far-reaching use
can be made of the data. Of these, the explanation of the experimental
evidence is the more important subject to be discussed, although the discus-
sion of theoretical considerations should not be omitted.

The experimental results of nuclear tests can be presented in a concise
form which differs from the laborious way in which they were obtained. An
example is the account by Snay-Butler (1957) on the underwater nuclear shock
wave. This short report covers the practical aspects of the free-water shock
wave. A similar account could be given for the shallow water shock wave, the
bubble, surface phenomena, etc., but readers may not find the answers to
their questions in such reports. In any comprehensive presentation of
underwater nuclear explosion phenomena, it is necessary to describe the
experimental results as well as what can be done in a case where the results
are not readily applicable. The presentation must also show how the answers
are obtained and to what extent the answers are re.i.. .'.. To accomplish
these objectives, theory and scaling analysis must be used.

Very few underwater nuclear test explosions have been conducted. Theory
must be used to make up for the lack of complete data. For a long time,
only one test provided data on the free-water shock wave. Commonly,
confirming tests are required before data of this kind are considered to be
reliable; however, considerable confidence has been placed in these data

because of the good agreement obtained between experiment and theoretical results. This reasoning, which considers theoretical results to be equivalent to missing confirming experiments, places a great deal of confidence on theory which may or may not be justified. Therefore, it is important to scrutinize such theories in order to see how much confidence they actually deserve.

The importance of theory becomes even more obvious if we realize that a number of effects, which are of military importance, cannot be readily measured in full scale tests, e.g., bubble phenomena. It will never be possible or justifiable to make so many nuclear tests as to render theoretical methods dispensable. In fact, those who have to take the responsibility of planning nuclear explosion tests must be particularly aware of the possibilities offered by theory or model testing. This is necessary to make sure that there is an actual need for full scale tests and that the information desired cannot be obtained by other means.

This does not mean that a discussion of underwater explosion phenomena should be a thesis on theoretical hydrodynamics; on the contrary, theoretical details should be omitted wherever possible. But the underlying basic concepts must be understood in order to appreciate the significance and the implications of the experiments.

1.2 THE TWO FACES OF THEORY. In some respects theory may be compared with, say, an electronic amplifier. Both are a maze of mysterious items, mathematical symbols in one case, tubes, wires, resistors, capacitors, etc., in the other.

When we use an amplifier, we seldom care about the design details but are mainly interested in what it can do, such as amplification, frequency response, and similar characteristics of performance. In the same way, the mathematical development of a theory is only a portion or one facet of the problem which concerns us. The most important question is:

> What can theory do for us?

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Often, little or no mathematics is needed to answer this question.

Many consider theory with a certain mistrust for which there are excellent reasons. Hydrodynamicists and other theoreticians, for instance, often surprise their listeners with bland statements that a particular problem can be readily calculated, whereas others cannot. Such statements reflect the correct and sometimes overlooked fact that certain problems are amenable to theory and others are not. (The same statement also holds true for experiments.) How can one judge whether or not a certain theory concerns a benign case?

It is our contention that this judgment requires almost no mathematical skills; it does require an understanding of a few basic physical concepts.

Neither theories nor amplifiers should be "black boxes" to those who use them. There should be an understanding of their basic principles, limitations, and abilities. We will attempt to further this understanding. It is almost superfluous to state that such an understanding is helpful not only in connection with theory, but also in the comprehension of the meaning of experimental results and of the nature of encountered phenomena.

1.3 APPROXIMATIONS. Calculations are sometimes made in which the problem is described in a thoroughly over-simplified manner. Often such calculations involve factors or parameters which must be estimated ("fudge" factors). These estimations are made in such a way that the results confirm existing experimental evidence. When applied to different conditions, completely meaningless data are sometimes obtained. Such calculations hardly deserve the name theory. On the other hand, the question of approximations is one which is of the greatest significance to us.

It is important to remember that any and every theory is more or less an appropriate approximation. This is because it is neither possible nor necessary to take all physical effects into account. The key to a successful theoretical study lies in the art of judging which approximations are acceptable and which are not.

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There are two types of approximation:

a) Approximations in the physical description of the problem.

b) Approximations made in the mathematical execution.

Although both types of approximation are of decisive importance for the success of a calculation, the physical approximations are of greater interest to us. If the physical basis of a theory is poor, the best mathematics cannot improve it.

1.4 SIMULATION OF NUCLEAR UNDERWATER EXPLOSIONS BY MEANS OF MODEL TESTS. A closely related field is that of model tests of nuclear explosions. Model tests are highly important tools which may successfully help to bridge the gaps in knowledge of many underwater nuclear explosion effects. No one should expect offhand that a chemical explosive charge fired underwater will simulate a nuclear explosion. When correctly planned and interpreted such tests can be of immense value, but there are cases where model tests are virtually useless.

Model tests and theory have one obvious fact in common: both are approximations of the real event. Whether they are excellent, fair, or unacceptable approximations depends on the circumstances. The understanding of the basic physical concepts will go far in permitting judgment on the quality of those approximations.

1.5 THE OBJECTIVE. It is our objective to summarize, in detail, the physical concepts which are important in the field of underwater explosions. This will be attempted in the simplest possible terms without undue sacrifice in rigor. The difficulties of such an attempt are only too well known. There is always the danger that some will object to the lack of strictness or that others will find the presentation too lengthy. Some readers may miss recipes for the quick calculation of underwater explosion phenomena. This paper is not written to provide such instructions. For instance, it is not our objective to describe the quantitative calculation

of the entropy increase in the shock front, but rather to show why such an increase takes place and what it means.

1.6 SUMMARY. The objective of this paper is to mediate between hydrodynamic textbooks and weapon effects reports. The intention is to further the understanding of the underlying basic principles of nuclear underwater explosions. Such an understanding will permit judgment on the trustworthiness of experimental results, of theories, and of model tests.

II. FLUIDS AND FLUID MOTION

2.1 FLUID MOTIONS OF EXPLOSIONS: WAVES AND MASS MOTION.

The study of explosion phenomena is a specialized and highly fascinating section of the physics of fluid motion. Although the term fluid motion may occasionally convey the idea of a steady flow, the field of fluid motion covers non-stationary problems. Among these are the rapidly changing and short-lived phenomena occurring in explosions.

A phenomenon of particular interest is the formation and propagation of waves. There are two distinct types of waves of concern: pressure waves and surface waves. The latter are of the familiar type of ocean waves with the exception that they are not formed by the wind but by the disturbances of the water surface caused by an underwater explosion. Pressure waves are the manifestation of the spreading of compression caused by the sudden action of the pressure of an explosion.

Waves, like any other flow phenomena, are processes of transport. It is a characteristic of wave motion that there is little or no significant transport of material: a wave transmits a state, such as the elevation of the water surface (Figure 2.1) or a rise of pressure, etc. Generally, waves are a mode of energy transmission whereby energy is transferred from one particle to the next. The speed of this transmission is called propagation velocity; the maximum value of the magnitude transmitted, the amplitude. Although the medium itself is not transmitted, it undergoes motions during the passage of a wave. The velocity of this motion is called the particle velocity.

The energy transmission in pressure waves is accomplished by the evident way that each particle exerts pressure upon the next and, in this fashion, passes a part or all of its energy to its neighbor. Another way of energy transmission which occurs in nuclear explosions is that caused by

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TIME t_1

PARTICLE "a"   PARTICLE "b"

TIME t_2

PARTICLE "a"   PARTICLE "b"
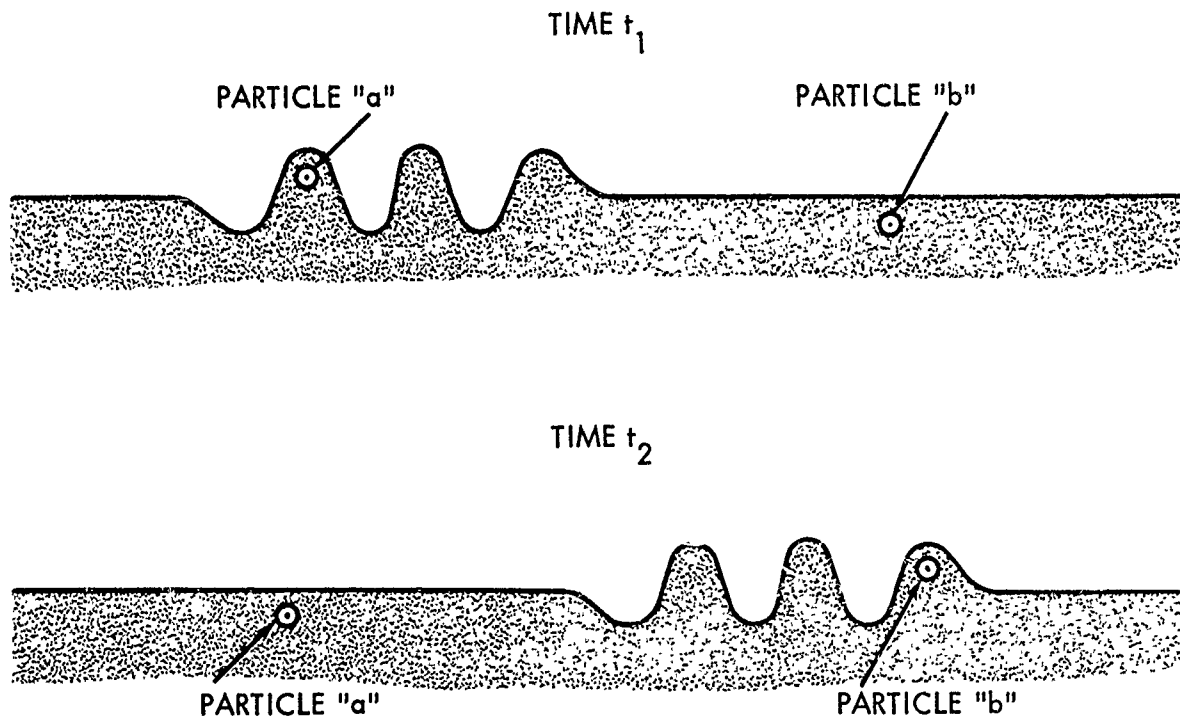
Figure 2.1 — Waves on the Water Surface.

If a stone is dropped into water, circular waves emerge from the point of disturbance. Omitting irrelevant details, the following is observed: In the above sketch the origin of the wave is far to the left-hand side. At the time t_1 the wave has reached particle "a" which is rising and falling while particle "b" is at rest. At the later time t_2 the wave has reached particle "b". Now particle "a" is at rest and often at the position it had before the arrival of the wave. There was a transport of energy but not of mass.

electromagnetic radiation. This phenomenon is very pronounced in nuclear explosions in air and also plays an important role in the formation process of the nuclear underwater shock wave.

Besides waves, underwater explosions also originate mass motions. These refer to a transport of material; for example, the pulsation and migration of the explosion bubble, the rising of the water column above the water surface, or the formation of craters in the bottom.

2.2 THE FUNDAMENTAL EQUATIONS OF FLUID DYNAMICS. Excluding the process of crater formation and other phenomena involving the bottom of the sea, all types of waves and mass motions in underwater explosions are, in principle, completely described by the pertinent solutions of the partial differential equations of fluid dynamics. These equations are the basis of all studies of fluid motion, not only for theoretical calculations, but also for the evaluation of experimental results and the design of model tests (scaling). These equations are derived from the principles of conservation of momentum, mass, and sometimes energy, as well as the second law of thermodynamics. Although the validity of these principles is beyond doubt, uncertainties arise owing to the need to approximate the mathematical treatment of these equations and because of insufficient knowledge of the behavior of the medium. It requires experience to choose the approximations which are simple enough for expedient treatment but do not unduly sacrifice accuracy.

2.3 EULER AND LAGRANGE EQUATIONS. There are two types of coordinate systems which can be used in these equations. One of these considers a fixed point of observation past which the medium flows with constant or variable speed. The equations which describe the flow seen from such a fixed point are the Euler equations.

The other form of equations, the Lagrange equations, use a coordinate system which moves with the fluid particle. For this purpose, it is

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necessary to identify the particle so that it can be distinguished from others; this is done by a coordinate sometimes called the "Lagrangian Label."

For the explosion phenomena this label is simply the position of the particle before the moment of explosion. In a nuclear underwater explosion, the particles adjacent to the bomb will be not only vaporized but also completely dissociated. These particles are subject to gamma, neutron, and other radiation and become radioactive. Particles at longer distances are dissociated, but do not become radioactive. At still greater distances the water is vaporized and, finally, particles further out are heated but remain liquid. All of these particles move, but the radioactivity and the entropy * (which dominates the tendency to ionization and vaporization) are unaffected by this motion. They are attached to each individual particle and move along with it. In such cases, Lagrangian coordinates are particularly convenient.

Although the use of the Lagrangian equations has distinct advantages in some cases, Euler's representation is preferable both from the mathematical and the physical point of view and is used in the majority of cases.

2.4 ON THE USE OF THE HYDRODYNAMIC EQUATIONS. We will not give the derivation of the equations of fluid dynamics since it can be found in any text of this subject. Instead, it will be attempted to acquaint the reader with the dominant role which these equations play in the study of fluid motions and with the methods of utilizing these equations.

It is important to point out that there are no general solutions of the fluid dynamic equations. This is not surprising for two reasons. First, these equations are of the non-linear type and therefore are not amenable to elegant mathematical treatment. Second, these equations describe such an

* See footnote on page 14.

immense manifold of different processes and flow patterns that such a solution, if it were possible, would be of extreme complexity.

Despite the absence of general solutions, the interpretation of these equations provides a wealth of information. Most of our theoretical discussions will be devoted to such interpretations. The reader will note that it is often possible to derive relatively simple formulations of various problems by means of a suitable and sometimes even highly elementary manipulation of these equations.

2.5 MODELS OF FLUIDS. There are several "models" of fluid motion, each of which is described (or defined) by a special set of partial differential equations. For our purposes, the important models are (a) the incompressible, non-viscous fluid motion, (b) the compressible, non-viscous fluid motion, and (c) the compressible and viscous fluid motion. These three possibilities are listed in order of increasing difficulties in the mathematical treatment. Since it is always desirable to use the most simple approach, it is of great importance to judge which of these models must be chosen. To do this, an understanding is needed as to what bearing the properties of fluids have on the phenomena to be studied.

2.6 PROPERTIES OF FLUIDS. "Fluid" is a broad term which covers any non-rigid medium – in particular, gases, vapors, and liquids. The concept of rigidity is closely related to that of shear strength. Consider a cube of any matter and assume that this cube is deformed into the shape shown in Figure 2.2 without changing the volume. We are making the assumption of constant volumes because every medium, even a gas, will react to a change of volume. This is a result of compressibility and not of rigidity.

The change of shape shown in Figure 2.2 will not be resisted by a fluid if it is performed slowly. However, a steel cube will strongly resist a transfer into such a configuration even if it is attempted at an extremely slow rate. Hence, steel and other solids exhibit a rigidity which fluids do not have.

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Figure 2.2 - Illustration of Rigidity.

This definition of rigidity may seem surprising, but it is the shear strength which prevents a vertical rod from falling apart as a liquid column does, or prevents craters from leveling out like a dent in the water surface. There are further differences, in particular the inability of gases and liquids to sustain tension. By tension we mean the opposite of compression, i.e., what engineers call tensile stress. It corresponds to a negative absolute pressure for liquids (Article 2.10) but cannot occur in gases. There are further differences between solids and fluids in the equation of state. These details go beyond the scope of interest of this study.

If the change of shape shown in Figure 2.2 is produced in a fluid within a very short time, i.e., at a high rate, there will also be a resistance. This effect is called the internal friction of fluids and it is closely related to the viscosity of fluids. In Figure 2.2, the cube may be visualized to consist of a great number of infinitely thin parallel layers. Internal friction is the resistance against the sliding of these layers.

2.7 <u>VISCOSITY</u>. This type of friction rarely occurs in underwater explosions because of the absence of a sliding motion. A fluid motion which would result in a deformation of particles as in Figure 2.2 is called a rotational motion. Most of the motions we are concerned with are of the irrotational form. However, there is another process where internal friction becomes significant.
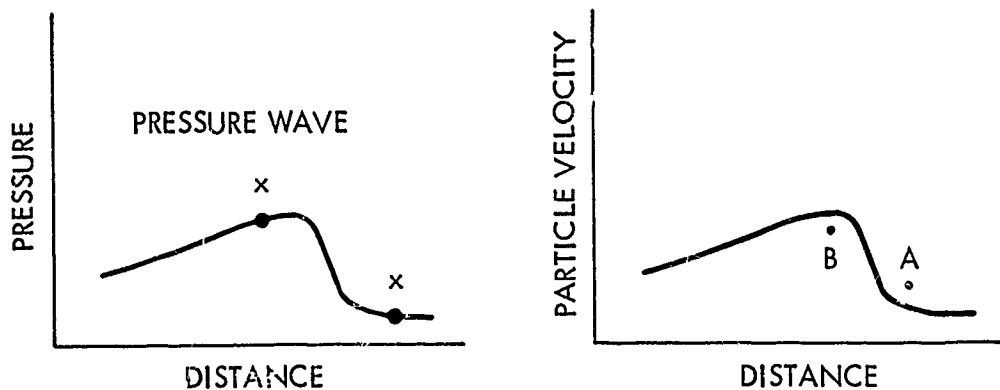
Figure 2.3 - Internal Friction in Pressure Waves.

In Figure 2.3 pressure and particle velocity of a pressure wave are plotted versus distance for a fixed moment of time. Internal friction occurs predominantly at all places where rapid changes of the velocity profile occur, or, to be more specific, at places where the radius of curvature of the curve is small, such as at the points A and B, or within the region x-x. This form of internal friction is analogous to that illustrated in Figure 2.2 but it is not as self-evident. An inspection of the pertinent hydrodynamic equation will demonstrate this phenomenon without difficulty. This equation (which

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expresses the principle of conservation of momentum) is for the case of viscous fluid motion

$$(2.1) \qquad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} ,$$

where u = particle velocity, ρ = density, P = pressure, ν = kinematic viscosity, x = distance, t = time. Equation (2.1) holds for a plane fluid motion. The equation is called the Navier-Stokes equation. (The Navier-Stokes equation incorporates certain approximations mentioned below, but these will in no way affect our conclusions.)

The term on the right-hand side accounts for the effect of viscosity. This term is inversely proportional to the radius of curvature[*] of the velocity profile which is in accordance with the statement made in connection with Figure 2.3. Equation (2.1) reduces to that of a non-viscous fluid if this term is omitted. There is no liquid or gas for which ν is zero. However, one may argue that, once the product of viscosity and the second derivative of the velocity profile is small in comparison with the other terms, the non-viscous fluid motion will be a good approximation of the real flow. Although this argument is valid, it is not without pitfalls, since it is often difficult to estimate the value of $\partial^2 u/\partial x^2$. A classical example is the steady flow across a cylindrical obstacle. The general flow pattern does not indicate the presence of large second derivatives in the velocity. Still, the non-viscous flow and that of a real fluid differ radically, even for a fluid of minute viscosity, because in the boundary-layer close to the wall $\partial^2 u/\partial x^2$ can reach very large values. In the study of underwater explosion phenomena, care must be taken to separate all those cases where inclusion of viscosity is important from those where it can be neglected. As will be seen, viscosity has very little effect on the fluid motion except in the shock front.

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[*] The radius of curvature of the u – x curve is

$$\left[1 + \left(\frac{\partial u}{\partial x} \right)^2 \right]^{3/2} \Big/ \frac{\partial^2 u}{\partial x^2} .$$

Here, the viscous fluid motion is accounted for by the Rankine-Hugoniot equations.

2.8 <u>IRREVERSIBLE AND REVERSIBLE PROCESSES</u>. Internal friction of a fluid, as any other form of friction, is an irreversible process. A part of the mechanical energy is transformed into thermal energy which leads to a heating of the fluid. According to the second law of thermodynamics, this thermal energy (or at least a part of it) cannot be returned into mechanical energy, and the original state cannot be attained again. Therefore, viscous fluid motion is an irreversible process or, in other words, the entropy[*] of the particles increases.

If we omit the right-hand term in Equation (2.1), i.e., if we consider a non-viscous fluid motion, we have a reversible or isentropic process. In this case every particle retains its value of entropy.

2.9 <u>THE EFFECT OF COMPRESSIBILITY</u>. Compressibility may be visualized as the resistance of the medium against a change of volume. It is well known that a gas can be compressed without building up high pressures, whereas high pressure is needed to change the volume of water. Hence, it might be said that gases are compressible, but water is almost incompressible.

The compressibility of the medium is an extremely important factor in explosion phenomena. Shock waves or blast waves which are produced by explosions are a direct consequence of the compressibility of the medium. In their nature, these waves resemble sound waves, the difference being in the amplitudes and their limited duration. The amplitudes of sound waves are infinitesimally small compared with those of explosion waves.

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[*] The concept of the entropy is often a stumbling block in the understanding of such processes. In Appendix B an attempt has been made to describe a few salient points of this concept as far as they are pertinent to our study.

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The velocity of propagation c of sound waves is defined by

$$(2.2) \qquad c^2 = \left(\frac{\partial P}{\partial \rho}\right)_S \qquad ,$$

where the subscript S indicates differentiation with constant entropy.

This magnitude is not directly applicable to the propagation velocity of high-amplitude waves, but plays an important role in the theory of these waves. In particular, c is a measure of the compressibility of a medium. (For most fluids, the sound velocity increases with pressure which indicates that the medium becomes less compressible as the pressure rises.) At atmospheric pressure and room temperature c = 332 m/sec for air and c = 1465 m/sec for water. The difference in these velocities is less than a casual observation of the compressibility of these media would seem to indicate. The reason for this is that at this point we consider the effect of compressibility in connection with pressure waves. In such a case the assumption of an incompressible medium is not appropriate for water; it would mean an infinitely high propagation velocity of sound as well as high amplitude waves. Nevertheless, interesting and useful attempts at an "incompressible" shock wave treatment have been made, e.g., Schauer (1948). Theories which are at such variance with the actual physical picture need careful scrutiny as to their validity and significance. Today, efforts of this type have been superseded by the advances in the theory of compressible fluid motion.

In the case of mass motions such as in bubble pulsations and in the case of surface waves, the term compressibility has a different meaning. It refers here to the change of the density (or volume) as a function of pressure. For water and moderate pressures, these changes are negligibly small. The incompressive fluid motion is a good approximation in such cases.

A quantitative measure of the effect of the compressibility in a flow process is the Mach Number

$$(2.3) \qquad M = \frac{u}{c} \qquad ,$$

where u is the particle velocity (for instance the maximum fluid velocity occuring in the phenomenon considered) and c the sound velocity. The case of "low" pressure is denoted by values of M which are small compared with unity, say M = 0.1. For such small Mach numbers, pressure waves can be approximated by the laws of sound waves (acoustic theory), and mass motions by the laws of an incompressible fluid.

2.10 LIQUIDS. In general, the laws of motion are the same for liquids and gases. Of course, the pertinent material constants must be used in either case. Phase changes, namely melting, freezing, or vaporization introduce additional problems, although the basic treatment remains the same. In underwater explosion phenomena, evaporation as well as freezing of water may occur. The latter may be caused by the high pressures of the shock wave and will be discussed in the section on the equation of state.[*] Water is vaporized near a nuclear underwater explosion, thus forming the nuclear explosion bubble.

Cavitation is a special form of evaporation caused by dynamic underpressures, such as those caused by the reflection of a pressure wave from the water surface. Cavitation is the result of the inability of water to withstand tension. Theoretically, clean and gas-free water can be subjected to a negative pressure equal to the cohesion pressure (which is about 3 kilobar = $4.3 \cdot 10^4$ psi) before "breaking," i.e., boiling. Experimental values depend on the degree of purity achieved for the liquid and the walls of the container; values up to 0.3 kilobar have been reported. Sea water always contains a great number of impurities which act as "cavitation nuclei." Since the surfaces of these impurities are often hydr(phobic (water repellent) there is little or no adhesion at the interface which could oppose tension. Therefore, bubbles are formed at such nuclei as soon as the pressure drops below either vapor pressure or the saturation

[*] See Chapter IV of the book "Underwater Nuclear Explosions."

pressure of the dissolved gases. The expansion of these bubbles prevents the occurrence of any substantial tension.

In principle, the hydrodynamic equations also describe cavitation processes, but the equations assume different forms for the cavitated and un-cavitated areas. Pressure waves in non-cavitated water, for instance, can be approximately described by the wave equation, see Appendix A. Wherever cavitation occurs, the wave equation is not applicable.

2.11 SOLIDS. Since the bottom of the sea affects underwater nuclear explosion processes, the properties of solid materials are of some concern to us. As explained above, there are two significant differences between a liquid and a solid: a liquid lacks shear strength and tensile strength whereas solids have both of these properties and, because of them, exhibit their characteristic behavior.

If we consider an irrotational fluid motion, no sliding (as depicted in Figure 2.2) occurs; therefore, shear strength does not come into play. If, in such a phenomenon, compression alone occurs, fluid motion will simulate the actual behavior of solids with acceptable accuracy. For example, in the treatment of shaped charge effects on steel plates, notable success was obtained by assuming that steel behaves like a heavy liquid. This treatment breaks down when the applied forces are less than the yield strength of the medium.

This treatment, the so-called liquid model, has been used in studies of underwater explosion phenomena. The model of a liquid bottom is useful in the description of the crater formation, the effect of the bottom on the pulsating bubble, and the bottom reflection of underwater explosion shockwaves.

When the pressure wave of an underwater explosion impinges upon the bottom, one part of the incident wave is reflected backward into the water, and another part enters the bottom (refracted wave). In a rigid bottom, the refracted wave commonly consists of a compression wave (P-wave) and a

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shear wave (S-wave). Both the P- and the S-wave carry energy down into the bottom. The energies of the reflected wave plus the refracted P- and S-waves are equal to the energy of the incident wave. If the amplitudes of the reflected wave are calculated, higher values are obtained for a liquid than for a rigid bottom, if all other parameters are the same. This is because the energy carried away by the S-wave is ignored. In many practical applications, however, the difference is not significant.

2.12 SUMMARY. Explosion phenomena are processes of fluid motion. They are described by the fundamental equations of fluid dynamics. The equations admit four models of a fluid, depending on the inclusion or omission of compressibility and viscosity. The term "fluid" designates any non-rigid medium: gas, vapor, or liquid. A fluid lacks shear strength. Viscosity or internal friction is the property of a fluid which opposes the rate of shear. Viscous fluid motion dissipates energy. An inviscid fluid motion is reversible (and vice versa). If the motion is irrotational, no shear deformation occurs.

Compressibility refers to the reaction against a change of volume. The Mach number is the measure of the effect of compressibility in a flow process. If the Mach number is small, the acoustic approximation is applicable to pressure waves and the theory of incompressible fluid dynamics is applicable to mass motions.

Although a liquid can theoretically be subjected to a tensile stress, an actual liquid, in particular sea water, "breaks," i.e., cavitates, almost immediately.

In some cases the motion of solids can be approximated by that of a liquid; for instance, the effect of the bottom of the sea can be studied by using the model of a liquid bottom.

III. THE SHOCK FRONT

3.1 <u>PROPERTIES OF A SHOCK FRONT</u>[*]. A shock front is a jump in pressure, velocity, density, and temperature. It occurs most often, but not always, when a pressure wave of high amplitude runs into an undisturbed medium. The medium is at rest up to the moment of the arrival of the shock front. When the shock front passes the point of observation, the pressure and the other magnitudes undergo a rapid, almost discontinuous, change. Behind this thin region of rapid rise, we have the beginning of the shock wave. In most cases the pressure of shock waves decreases behind the shock front. The characteristic difference between the front and the subsequent wave is the rate of change in pressure, velocity, and density. This rate of change is extremely rapid within the front, but comparatively slow in the wave which follows.

In an era where problems of supersonic flight and atomic explosions are widely discussed, shock waves appear to be a familiar subject to everyone. Few realize that a shock wave is a rather unusual physical phenomenon. Although everyone is willing to grant that jumps rarely occur in nature on a macroscopic scale, a discontinuous rise seems to be entirely acceptable for a shock wave. Actually, a shock front is strictly not a discontinuity and the steep rise of pressure at the front of shock waves is a result of specific circumstances which are significant not only for the theory of shock waves, but also for a general understanding of their behavior.

The following development of the fundamental properties of a shock front together with an historical account, is not done for the sake of literacy.

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[*] In this Article, energy transfer by radiation processes is excluded.

It is interesting as well as revealing to note that eminent scientists in the past have had difficulties in visualizing and understanding this phenomenon. Analyzing these difficulties in retrospect will give us a clear insight into the nature of the problem.

3.2 PRESSURE WAVES. Newton (1713) was the first to attempt to analytically describe pressure waves in a compressible medium. He derived an expression for the propagation velocity of sound in air. This equation was based on the assumption that the change of density is proportional to the change of pressure, or in thermodynamic terms, on an isothermal change of state. Inserting the proper magnitudes into his equation yielded

Sound velocity in air = 280 meters/sec.

This velocity is now known to be too low. Laplace (1816) improved Newton's formula by the introduction of the isentropic which was previously called the adiabatic change of state. His improved formula yielded

Sound velocity in air = 332 meters/sec.

This value agrees with the best measurements known today and Laplace's formula is one of the classic triumphs of theoretical physics. The confidence in Laplace's formula has become so great that his formula is being used to determine the isentropic exponent of gases by measuring sound velocity. This relationship brought about a change in thinking concerning the effect of heat conduction. Newton's equation is in accord with the accepted fact that any rise in temperature will be reduced and finally equalized by heat conduction. Laplace's result established that heat conduction can be neglected for rapid temperature changes. In fact, pressure waves in compressible media constitute one of the rare examples where an isentropic change occurs in nature. Stokes and Lord Rayleigh further firmed this thinking by including viscosity and heat conduction in this relationship. For extreme conditions

at very high frequencies, with rapid changes in amplitude, it is necessary to correct for these effects. Excluding such extreme conditions (which obviously prevail at a shock front), the isentropic change of state is appropriate for pressure waves. This means that such waves are reversible processes.

3.3 RANKINE WAVES. At the time when this line of thinking was firmly established in 1870, Rankine derived his renowned relationships for a "stationary wave," as he called it. It was not Rankine's intention to consider a shock front, but rather a plane wave of high amplitude which does not undergo changes as it propagates. Figure 3.1 illustrates the situation.
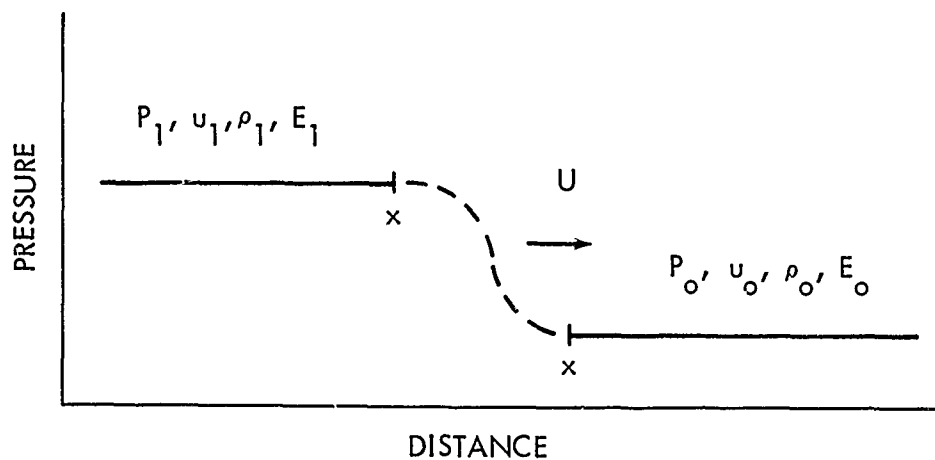

Figure 3.1 - Rankine Wave.

The wave is assumed to propagate from left to right with the velocity U. In the general case, the medium ahead of the wave may not be at rest. Before arrival of the wave there is a density ρ_o, a pressure P_o, a particle velocity u_o, and internal energy E_o. For Rankine's investigations, details of the increase of pressure and other magnitudes do not matter, i.e., the structure of the dashed portion x-x in Figure 3.1 is irrelevant. The essential point is that the pressure P_1, particle velocity u_1, and density ρ_1 behind the increase remain constant. Application of the principles of the conservation of mass, momentum, and energy yield the well-known relations

$$(3.1) \qquad \rho_1 (U - u_1) = \rho_o (U - u_o)$$

$$(3.2) \qquad \rho_o (U - u_o) u_1 = P_1 - P_o$$

$$(3.3) \qquad E_1 - E_o = \frac{P_1 + P_o}{2} \left(\frac{1}{\rho_o} - \frac{1}{\rho_1} \right)$$

$$= \frac{P_1 + P_o}{2} (v_o - v_1).$$

Here E denotes the internal energy and v, the specific volume $v = 1/\rho$.

Nineteen years after Rankine, Hugoniot published essentially equivalent relations. Obviously, he had derived these independently of Rankine. It has become customary to call equations (3.1) to (3.3) the Rankine-Hugoniot relations.

3.4 THE RANKINE-HUGONIOT ADIABAT. Of particular significance is the energy equation (3.3) which is often called the Rankine-Hugoniot adiabat. This equation resembles that for an isentropic change of state, which for infinitely small changes of pressure or density is given by

$$(3.4) \qquad dE = - Pdv$$

whereas (3.3) amounts to

$$(3.5) \qquad\qquad \Delta E = - P_{average} \Delta v.$$

For infinitely small pressure changes, the isentropic (3.4) and the Hugoniot adiabat (3.5) coincide. Of course, for finite amplitudes, for which (3.5) has been specifically derived, the isentropic (obtained by the integration of (3.4)) differs from the Rankine-Hugoniot adiabat. The most significant difference is that the entropy is not constant along the Rankine-Hugoniot adiabat (3.3).

Hence, a Rankine wave is necessarily an irreversible process. This was recognized by Rayleigh 40 years after Rankine's publication. Rayleigh pointed out that the Rankine wave is physically possible only for compressions, but not for expansions in which the entropy would decrease. Rayleigh further pointed out that at the front of a Rankine wave a dissipative process must occur within the region of the pressure increase. This was in 1910, just before the First World War. At this time many workers did not believe that a Rankine wave would occur in nature. As late as the 1932 edition of Lamb's classical "Hydrodynamics", doubt was cast on the validity of the Rankine-Hugoniot relations, stating: "no physical evidence is adduced in support of the proposed law."

3.5 RIEMANN WAVES. This feeling of doubt becomes understandable if we realize that long before Rankine, Riemann (1860) established his famous relationships for high-amplitude waves (described in Article 4.3), which seemed to include the case of a stationary wave. But in contrast to Rankine's result, Riemann's equations involved an isentropic change of state which, in the line of thinking of that time, may have appeared to be more reasonable and acceptable. On the other hand, application of Riemann's propagation theory to a stationary wave leads immediately to a discrepancy shown in Figure 3.2.
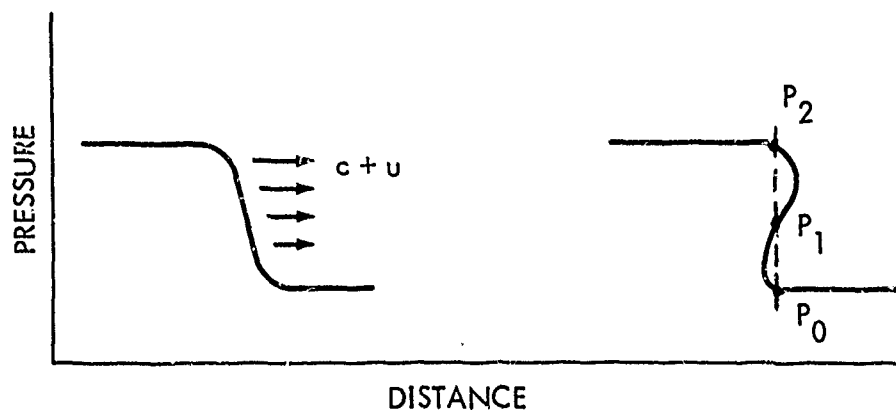
Figure 3.2 - Step Wave (According to Riemann's Theory).

Assuming a plane and stationary wave, Riemann's theory can be interpreted that each point of the wave travels with the velocity $c + u$. Points behind the head of the wave remain stationary because the pressure and, therefore, c, as well as u, is constant. However, the head of the wave does not remain stationary. Since $c + u$ increases with increasing amplitude, points of the head which have a higher pressure move faster than those of lower pressure. Theoretically, this would lead to the configuration of the head (shown in Figure 3.2) which, at the right-hand side, depicts the same wave for a later moment of time. Such a configuration would mean that three different pressures occur at the same time and at the same place (namely, P_0, P_1, and P_2), which is physically impossible.

Earnshaw reached a similar conclusion in 1858 on the basis of a mathematical manipulation of the hydrodynamic equations, namely, that a stationary pressure wave of high amplitude is incompatible with an isentropic change of state.

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3.6 THE SHOCK WAVE PARADOX. Since the mathematical validity of Rankine's as well as Riemann's theory was beyond doubt, the workers of these times were confronted with a highly perplexing paradox which probably induced many of them to let the matter rest.

Today, we understand that the prerequisite for a Rankine wave is an energy dissipation within the region of the rise. The pressure curve dashed in Figure 3.1 must be just so steep that the energy dissipation due to viscosity is such that the Rankine-Hugoniot adiabatic (3.3) is satisfied.

The Rankine-Hugoniot equations refer to the motion of a viscous fluid. The circumstance that viscosity does not appear in these equations must not obscure this fact. It is not necessary to know these magnitudes, if – as in the Rankine-Hugoniot treatment – only the initial and final state of the process is considered. If it is desired to calculate the details of the pressure rise, viscosity and heat conduction must be explicitly introduced.

It was not until the end of World War I that the existence of shock waves was established by several workers such as Ruedenberg (1916), Becker (1922), and notably Stodola (1924) who produced experimental evidence of shocks in steam nozzles. It had taken almost half a century until the correct interpretation of Rankine's equations was achieved.

We now return to Article 2.7 where the effect of viscosity on the shock front was discussed. A fluid with zero viscosity is called ideal. Such fluids do not exist and there is a difference between an ideal fluid and a real fluid of vanishing, but finite, viscosity. Figure 3.2 is a correct representation of a "shock" wave in an ideal fluid. It represents a physical unreality, since three different pressures cannot occur at the same time and the same place. The exactly analogous, but less striking evidence of ideal fluid behavior is d'Alembert's paradox (1744): the drag of a body
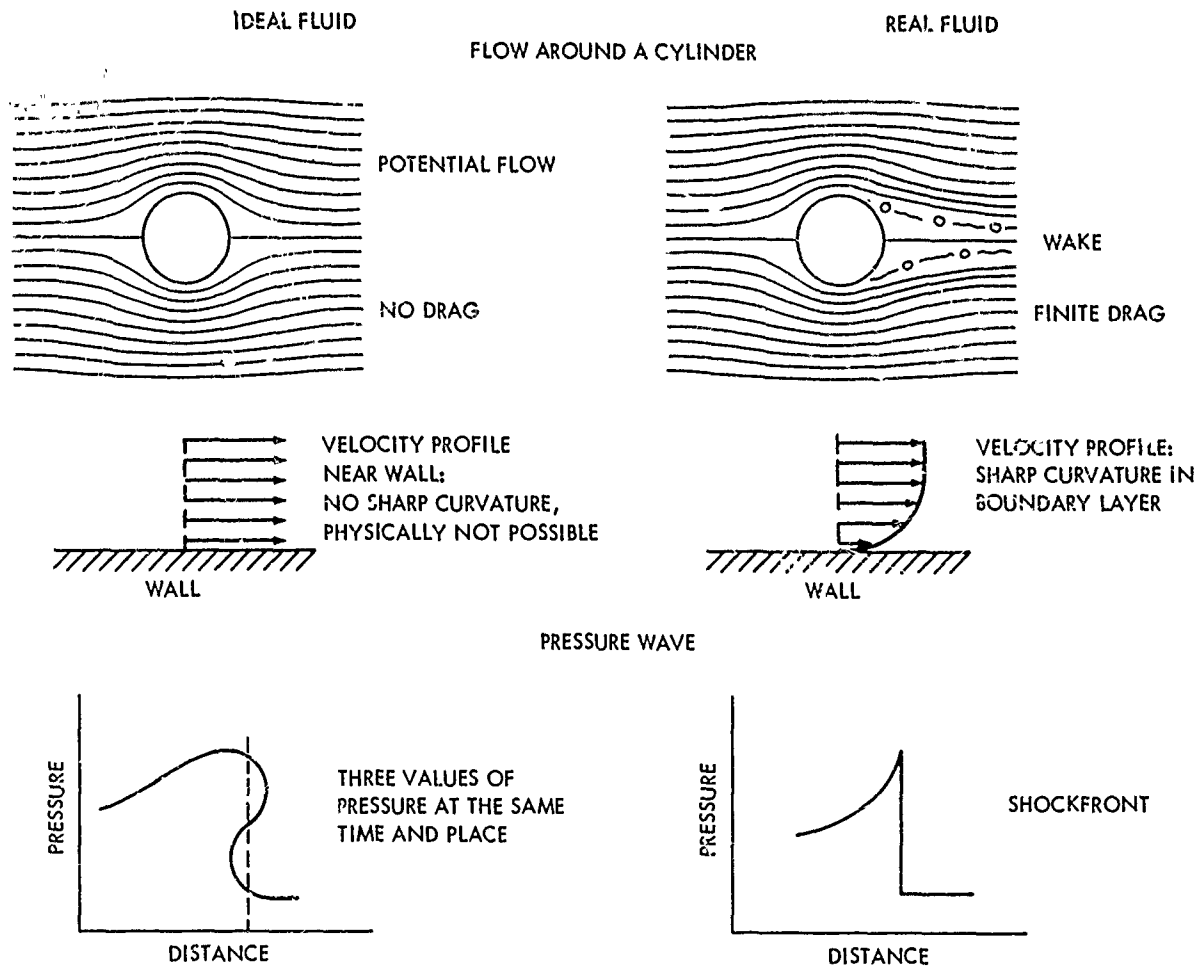
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in an ideal fluid is zero. Both d'Alembert's paradox and the shock wave paradox show one and the same thing, namely the limited scope of the ideal fluid model. Real fluids, even if their viscosity is exceedingly small, do not lead to these paradoxes: there is (a) a boundary layer around the body (Prandtl, 1904) and (b) a shock front at the head of the pressure wave. These paradoxes are illustrated in Figure 3.3.

The flow is irreversible and rotational within the boundary layer and shock front. Outside these regions the assumption of an ideal fluid, i.e., of a reversible and irrotational flow, gives reliable results.

It is easy to see in hindsight that these intricacies of the ideal fluid model played a big role in the difficulties which the 19th century had in this field. The clarification of these paradoxes and the understanding of the scope of the ideal fluid model is an achievement of modern 20th century physics, although it is less glamorous than those in other sections of this science.

3.7 THE THICKNESS OF THE SHOCK FRONT. This clarification did not conclude the arduous process of obtaining a grasp of the nature of the shock front. A new stumbling block arose - the thickness of the front. As shown in Figure 2.3, the thickness of the front is that region where the effect of viscosity is predominant, i.e., the region x-x. Turning to Figure 3.1, the thickness of the front x-x designates the closest points of a step wave between which the Rankine-Hugoniot conditions are applicable. Calculations by Becker and others indicated such an extremely steep rise within the shock front that the thickness of the front would be something like the mean free path of a molecule. It took about 30 years more to explain and correct this controversial result.

Truesdell (1952), Gilbarg-Paolucci (1953), and Lighthill (1956) finally found that the Navier-Stokes equation is not suitable for these calculations and must be replaced for dynamical problems by another equation which

IDEAL FLUID
REAL FLUID
FLOW AROUND A CYLINDER

POTENTIAL FLOW

NO DRAG

WAKE

FINITE DRAG

VELOCITY PROFILE
NEAR WALL:
NO SHARP CURVATURE,
PHYSICALLY NOT POSSIBLE

WALL

VELOCITY PROFILE:
SHARP CURVATURE IN
BOUNDARY LAYER

WALL

PRESSURE WAVE

PRESSURE

DISTANCE

THREE VALUES OF
PRESSURE AT THE SAME
TIME AND PLACE

PRESSURE

DISTANCE

SHOCKFRONT

The model of an ideal fluid is an important tool of fluid
dynamics. The model fails and leads to paradoxes when
the second derivative of the velocity with respect to
distance becomes predominant.

Figure 3.3 — Behavior of Ideal and Real Fluids.

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omits some terms in the Navier-Stokes equation and introduces others. On the basis of these investigations, it has now been established that the rise within a shock front is continuous, but steep. The transition takes place within a distance which is a multiple of the mean free path of a molecule, and depends on the strength of the shock wave as well as on the distance traveled. The thickness of underwater explosion shock fronts is very small near the point of explosion and increases gradually as the wave travels away.

To Summarize: The shock front comprises a region of finite thickness. The Rankine-Hugoniot relations hold for the conditions at the beginning and at the end of that region, i.e., for x,x in Figure 2.3. Note that, strictly speaking, .he shock wave peak pressure does not necessarily coincide with the pressure P_1 behind the front occurring in the Rankine-Hugoniot equations. This is illustrated in Figure 2.3: P_1 would occur at the left-hand x. However, for almost all practical purposes in explosion research this distinction is a trivial one and may be ignored. Furthermore, for almost all problems in weapons effect studies the shock front may be regarded as an infinitely thin region, i.e., as a discontinuity. (In the latter case, shock waves relatively close to the point of explosion are of interest.)

3.8 EQUATIONS DESCRIBING SHOCK FRONT PARAMETERS. In the following, a few of the important relations which can be derived from the Rankine-Hugoniot equations are summarized. The symbols are explained in Figure 3.1 and Article 3.3.

Propagation velocity of the shock front

(3.6)
$$U = u_0 + v_0 \sqrt{\frac{P_1 - P_0}{v_0 - v_1}} \ .$$

Particle velocity of the shock front

(3.7)
$$u_1 = u_o + \sqrt{(P_1 - P_o)(v_o - v_1)} \; .$$

The following combinations of these equations are sometimes useful

(3.8)
$$\frac{U - u_1}{U - u_o} = \frac{\rho_1}{\rho_1 - \rho_o} \quad ; \quad \frac{U - u_o}{U - u_1} = \frac{\rho_1}{\rho_o} \; .$$

A magnitude of great importance in shock wave calculations is the sound velocity c, defined by

(3.9)
$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_S = -v^2 \left(\frac{\partial P}{\partial v}\right)_S \; .$$

There is no simple relationship of general validity for the sound velocity behind the shock front, c_1, in terms of other shock front parameters. The sound velocity must be calculated together with the Rankine-Hugoniot adiabat using standard thermodynamics and equation of state data.

Closed expressions can be obtained if the medium has the properties of an ideal gas. In this case

(3.10)
$$E = \frac{Pv}{\gamma^o - 1} \quad \text{and}$$

(3.11)
$$c^2 = \gamma^o Pv \; ,$$

where γ^o is the isentropic exponent. Introduction of (3.10) into (3.3) gives the Rankine-Hugoniot adiabat for an ideal gas

(3.12)
$$P_1 = P_o \frac{(\gamma^o + 1) v_o - (\gamma^o - 1) v_1}{(\gamma^o + 1) v_1 - (\gamma^o - 1) v_o}$$

$$(3.13) \qquad v_1 = v_0 \frac{2\gamma^0 P_0 + (\gamma^0 - 1)(P_1 - P_0)}{2\gamma^0 P_0 + (\gamma^0 + 1)(P_1 - P_0)} \ .$$

For comparison we note that the true adiabat or isentrope reads

$$(3.14) \qquad P_1 = P_0 \left(\frac{v_1}{v_0}\right)^{\gamma^0} \ .$$

For small differences between P_1 and P_0, both adiabats (3.12) and (3.14) coincide, but they exhibit an entirely different behavior for large pressure differences.

The entropy increase across the shock front for an ideal gas is

$$(3.15) \qquad S_1 - S_0 = c_v \ \ln \frac{P_1}{P_0} + c_p \ \ln \frac{v_1}{v_0} \ ,$$

where c_p and c_v are the heat capacities at constant pressure and volume, respectively.

3.9 GAMMA FOR IDEAL GASES AND FOR REAL MEDIA. It must be stressed that the relations (3.10) through (3.15) hold for ideal gases only. For an ideal gas the relation $\gamma^0 = c_p/c_v$ is valid. For an imperfect medium, water in particular, the ideal gas relations are not applicable. Then, γ is not c_p/c_v but is given by a more general equation, which for ideal gases reduces to this ratio. For a non-ideal medium, the ratio c_p/c_v has no practical thermodynamic significance related to fluid mechanics. It must be noted that the general definition of γ in fluid dynamics is

$$(3.16) \qquad \gamma = \frac{c^2}{Pv} = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S = -\left(\frac{\partial \ln P}{\partial \ln v}\right)_S \ .$$

For an imperfect medium, equation (3.10) takes the form (Snay, et. al. 1956)

$$(3.17) \qquad \gamma = \frac{Pv}{E} + 1 + \left(\frac{\partial \ln E/Pv}{\partial \ln v} \right)_S .$$

This expression reduces to equation (3.10) if the last term vanishes. Although not strictly valid, the ideal gas relation (3.10) has been applied to the highly compressed and imperfect gaseous detonation products of explosives, Jacobs (1956). Such approximations can be of great value. For water, no attempt of this kind has so far met with notable success.

3.10 HIGH PRESSURE REGION OF THE RANKINE-HUGONIOT ADIABAT. Ideal gas relations may become applicable at extreme temperatures where the medium is completely dissociated and ionized. Several workers have treated such a plasma as an ideal monatomic gas for which $\gamma^O = 5/3$.

If the pressure P_1 in the Hugoniot relation (3.13) is increased to large values, the specific volume becomes

$$(3.18) \qquad \lim_{\substack{v_1 \to v_o \\ P_1 \to \infty}} v_o \frac{\gamma^O - 1}{\gamma^O + 1}$$

$$\to 0.25 \, v_o \qquad \text{for a monatomic ideal gas}$$

and it is seen that in a Rankine-Hugoniot process, the density $\rho_1 = 1/v_1$ cannot be increased by high pressures beyond a certain value, such as $4 \cdot \rho_o$, if $\gamma^O = 5/3$.

The ideal gas approach is justified for exceedingly high temperatures and moderate densities. For conditions, as they occur in underwater nuclear explosions, the electrons may not behave like an ideal gas, but like a

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degenerate Fermi-Dirac electron gas, see Latter and Latter (1955). Figure 3.4 illustrates the various possible forms of adiabats for water.

3.11 EQUATIONS FOR LOW PRESSURE. It is also possible to derive relations for the low pressure end of the Rankine-Hugoniot adiabat. Kirkwood-Montroll (1942) obtained the following power series applicable to low pressures:

$$(3.19) \qquad v = v_o \left(1 - \frac{p}{\rho_o c_o^2} + \frac{\kappa \rho_o}{4} p^2 \ldots\right)$$

$$(3.20) \qquad U - u_o = c_o \left(1 + \frac{\kappa \rho_o^2 c_o^2}{4} p \ldots\right)$$

$$(3.21) \qquad u_1 - u_o = \frac{p}{\rho_o c_o} \left(1 - \frac{\kappa \rho_o^2 c_o^2}{4} p \ldots\right)$$

$$(3.22) \qquad c = c_o \left(1 + \left(\frac{\kappa \rho_o^3 c_o^4}{2} - 1\right) \frac{p}{\rho_o c_o^2} \ldots\right)$$

$$(3.23) \qquad S_1 - S_o = \frac{\kappa}{12 T_o} p^3 \ldots \ldots ,$$

where $\kappa = \left(\dfrac{\partial^2 v}{\partial p^2}\right)_S = 0.017$ cc/kilobar^{2}gm

c_o = sound velocity)   in the medium
)   before the
T_o = temperature)   shock front

p = excess pressure = $P - P_o$.

For approximate calculations, the following numerical values may be used: $c_o = 1480$ m/s and $\rho c_o^2 = 22$ kilobar. These values hold for fresh water at a temperature of 20°C and sea water of 8°C; conditions which often

prevail in experimental testing or in the Atlantic. The corresponding values of v_o are 1.0002 gm/cc and 0.976 gm/cc, respectively. For more detailed information, a paper on the equation of state of water should be consulted.

Of particular importance is relation (3.23) which shows that for low pressures, the entropy increase is proportional to the cube of the pressure amplitude, p. This means that the entropy increment is small for low pressures. Hence, for vanishing amplitudes, the Rankine-Hugoniot adiabatic becomes a reversible process. This is one of the justifications of the "acoustic" approximation for shock waves which will be further discussed in Article 4.5 and in Appendix 1. Furthermore, equation (3.20) shows that the propagation velocity of the shock front running into a still medium ($u_o = 0$) approaches the speed of sound for vanishing pressure.

Although it is possible to make statements about the low pressure and very high pressure region of the Rankine-Hugoniot adiabat, the entire intermediate region must be laboriously calculated on the basis of P-v-T data for water. Details go beyond the scope of this paper.

Figure 3.4 illustrates the two adiabats, the Rankine-Hugoniot curve and the isentrope.

3.12 <u>ENERGY DISSIPATION AT THE SHOCK FRONT</u>. We have seen that a prerequisite of the existence of a shock front is that there is a certain amount of irreversible energy dissipation. A conspicuous evidence of this energy dissipation is that the temperature of a particle has increased from T_o to T_{oi} when the shock wave has passed over it and after the pressure has returned to its original value. To heat the particle a certain amount of energy is required. Since we consider a process where the initial and final point are at the same pressure, namely the hydrostatic pressure of the undisturbed water, then this energy corresponds to an increase of the heat contents at constant pressure, i.e., of the enthalpy H which is simply
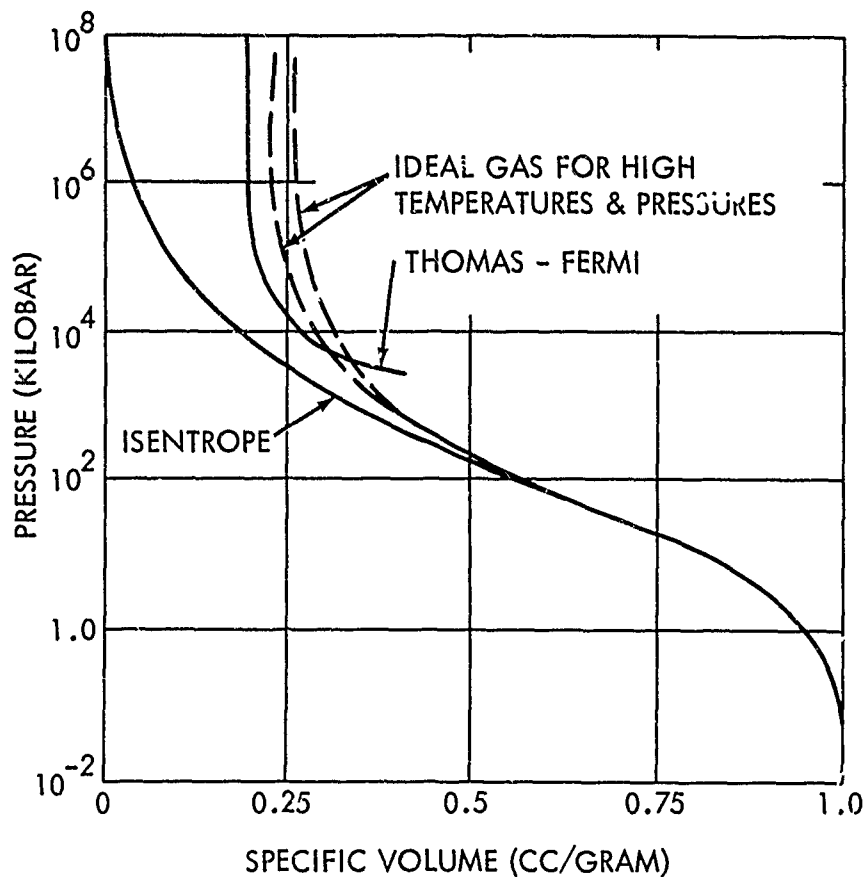
Figure 3.4 - Isentrope and Rankine-Hugoniot Adiabat for Water
(Schematic)

Isentrope and Rankine-Hugoniot adiabat coincide for low pressures.
The Rankine-Hugoniot curve as an asymptote for v = 0.25 gram/cc, if ideal
gas relations are assumed for high pressures and temperatures, implying com-
plete dissociation and ionization as well as non-degenerate electrons. De-
pending on the variation of γ along the curve, the asymptote may be
approached from either side.

If a degenerate electron gas is assumed, the Rankine-Hugoniot curve
approaches an asymptote resulting from the Thomas-Fermi model for highly
compressed media.

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(3.24) $\qquad H_{oi} - H_o = h = c_p (T_{oi} - T_o),$

where c_p is the heat capacity at constant pressure. The magnitude h is called "the dissipated enthalpy increment." Another term occasionally used is "waste energy." The expression for h can be readily obtained from (3.3) and (3.4) with the use of the thermodynamic identity $H = E + Pv$:

(3.25) $\qquad h = \dfrac{P_1 - P_o}{2} (v_o + v_1) - \displaystyle\int_{P_o}^{P_1} v(P, S = const)\, dP.$

Figure 3.5 illustrates the meaning of this relation in a P-v diagram. The curve extending from the point A, (P_o, v_o), to point B, (P_1, v_1), is the Rankine-Hugoniot adiabat. The dashed curve between B and D, (P_o, v_{oi}), is the true adiabat, i.e., the isentropic. The compression, which a fluid particle undergoes when the shock front passes over it, is described by the Rankine-Hugoniot curve, whereas the expansion behind the shock front follows the isentropic curve. The increase of the specific volume $v_{oi} - v_o$ is the consequence of the heating of the water particles by the irreversible processes occurring in the shock front.

The diagram of Figure 3.5 allows for the following interpretations: The heat content at the point P_1, v_1 is

(3.25a) $\qquad H_1 - H_o = \dfrac{P_1 - P_o}{2} (v_o + v_1) =$ area of trapezoid $A\,B\,B_o\,C_o$.

This constitutes the first term of equation (3.25). The second term of this equation is

(3.25b) $\qquad H_1 - H_{oi} = \displaystyle\int_{P_o}^{P_1} v(P, S=const)\, dP =$ area $D\,B\,B_o\,C_o$ bound by the dashed curve.
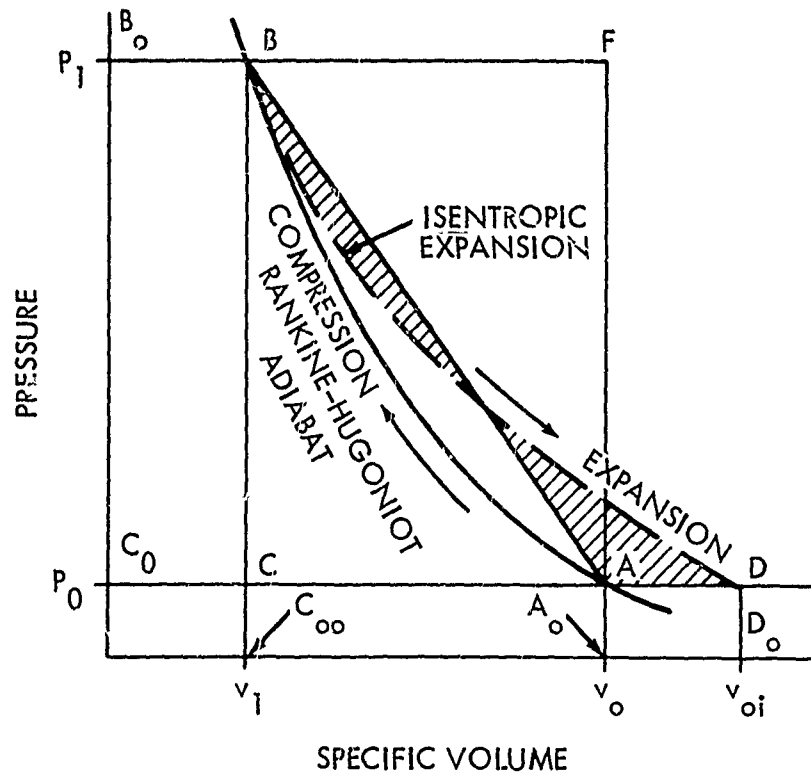
Figure 3.5 -- Interpretation of the Dissipated Energy.

When fluid is compressed in a shock front, the pressure P changes as a function of the volume v according to the Rankine-Hugoniot relation. Expansion occurs along the isentrope. Thus, the original state, P_o, v_o, is not obtained after the gas has re-expanded to P_o: the process is irreversible.

This diagram allows for geometric interpretations which are due to Porzel (1956).

Thus, h, which is the difference between these two terms, is the shaded area in Figure 3.5.

There are further interpretations which are interesting, but not always useful. According to (3.3) and (3.7)

$$(3.26) \qquad E_1 - E_0 + u_1^2/2 = P_1(v_0 - v_1) \quad \text{for } u_0 = 0$$

which is the area of the rectangle $A_0\ C_{oo}\ B\ F$. Thus, this area represents the total energy of the particle behind the shock front. The kinetic energy of this particle is given by

$$(3.27) \qquad u_1^2/2 = (P_1 - P_0)\ (v_0 - v_1)/2,$$

hence the area of the triangle ABC.

A crude approximation for h can be obtained by ignoring the difference $v_{oi} - v_o$, i.e., letting the points A and D coincide (Porzel 1956). Then h is equal to the lens-shaped area between the Rankine-Hugoniot curve and the chord AB. For water and moderate amplitudes, this approximation is not objectionable. It breaks down, as its originator has clearly stated, when the energy-dissipation is so large that water is vaporized after the passage of the shock wave. Then, v_{oi} and v_o differ so much that the accuracy of the above approximation becomes unacceptable. Since relatively accurate equation of state data are available for water in the low and medium pressure range, h can be calculated directly without recourse to this approximation.

3.13 ENTROPY AND DISSIPATED ENTHALPY. The magnitude h is closely connected with the entropy increase within the shock front: h would be zero if the shock adiabat were an isentrope. The interrelationship between entropy and dissipated enthalpy is given by (Snay, et. al. 1956)

$$h = \int_{S_o}^{S} T\,(S,\ P = P_o)\,dS$$

(3.28)

$$= \bar{c}_p\,T_o\,\left(\exp\left\{\frac{S-S_o}{\overline{c_p}}\right\} - 1\right),$$

where T_o and S_o are the temperature and entropy respectively before the passage of the shock front. The magnitudes $\bar{c}_p$ and $\overline{c_p}$ are the mean and logarithmic average heat capacity respectively:

(3.29a)

$$\bar{c}_p = \frac{\displaystyle\int_{T_o}^{T} c_p\,dT}{T - T_o}$$

(3.29b)

$$\overline{c_p} = \frac{\displaystyle\int_{T_o}^{T} c_p\,\frac{dT}{T}}{\ln(T/T_o)}$$

For water, $\bar{c}_p$ and $\overline{c_p}$ differ only slightly from the actual heat capacity c_p.

It has been shown in the preceding Article that for low amplitudes the entropy increment is proportional to the cube of the excess shock pressure p. From (3.28) one finds for small entropy changes with (3.23)

(3.30)

$$h = \frac{\kappa}{12}\,p^3,$$

where κ is explained in Article 3.11.

3.14 NUCLEAR BUBBLE FORMATION AND SHOCK FRONT PROCESSES.
Commonly shock wave and bubble phenomena are regarded and treated as distinctly separate phenomena. However, there is an intimate relationship

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between these two processes for a nuclear explosion in spite of their remoteness in space and time.

It is a well known feature of conventional underwater explosions that the gaseous reaction products of the explosive form a bubble which oscillates. Nuclear underwater explosions produce similar bubble phenomena, but the nature of the bubble formation is different.

The formation of the high explosive bubble is often attributed to the expansion of the gaseous reaction products which push the water away from the center of the explosion. Such a mass motion overshoots the point of pressure equilibrium and leads to pulsations of the bubble. This explanation is not entirely precise: the pressure of the explosion products forms, first of all, the shock wave. In the wake of such a spherical pressure wave, a flow process, "the afterflow," is originated which results in an outward motion of water. Such an outward flow must necessarily produce a cavity (see Article 4.11).

If a pressure wave could be generated in a body of water without an exploded charge or a transducer at the center, the outward mass motion would cause cavitation and, thus, a cavitation bubble. The presence of gaseous explosion products precludes cavitation since the expanding gases fill the cavity. The ability of the gases to expand not only permits this cavity to grow, but also enhances the rate of growth because of their initially high pressure. (Here, the above concept, which visualizes the gases pushing the surrounding water outward, applies, but this concept describes only a part of the process.)

Nuclear bubbles are filled with steam as well as dissociated and ionized water. It is often said that the "tremendous" heat of a nuclear explosion vaporizes and ionizes the ambient water. This explanation is objectionable because it will be almost always interpreted to mean that radiation produces this effect. Only about one thousandth of the mass of the gaseous contents in a

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nuclear bubble is formed by radiation, while the rest is formed by the energy
dissipation at the shock front.

An underwater nuclear explosion resembles the above case of a spherical
pressure wave in a space consisting exclusively of water. The nuclear device
is so small when compared with the equivalent charge of common explosive
that its presence can be neglected. Radiation will immediately form a fire
ball which may be considered as the origin of the nuclear explosion bubble.
But, its dimension is small compared with the size of the chemical charge and
this process has little bearing on the nuclear bubble formation.

As the shock wave emerges from the fireball each water particle under-
goes an entropy change when the shock front passes over it. For very high
amplitudes, the particle is immediately dissociated by the high temperature
of the shock. At lower amplitudes, i.e., farther away from the fire ball,
the water molecule remains intact. It is not a proper question to ask as to
whether or not this particle is in the liquid cr gaseous (vapor) state because
at the exceedingly high pressures and temperatures which prevail at this
moment such a distinction cannot be made. (Only for pressures below critical
may one speak of liquids or vapors.) The point is: will this particle be in
the vapor or in the liquid state after it has isentropically expanded to the
pressures prevailing in or near the bubble?

The answer to this question can be readily obtained from the "steam
tbles" found in any book on engineering thermodynamics. Figure 3.6 is
taken from such tables. For instance, if a particle suffers an entropy
increase of $\Delta S \sim 1.05$ BTU/$^\circ$F lb, half of its mass will be vaporized, the
other half will remain liquid. (This ratio is approximately independent of the
pressure.) Suppose the pressure of the particle has dropped to just one
atmosphere after passage of the shock wave. For this pressure, a particle
is completely vaporized, if $\Delta S = 1.76$ BTU/$^\circ$F lb. It remains liquid, i.e.,
it is at its boiling point, for $\Delta S = 0.312$ BTU/$^\circ$F lb. These entropy increments

depend only upon the shock front pressure and are listed in most tables on shock parameters for water. They are plotted in Figure 3.7. The graphs of Figures 3.6 and 3.7 permit the following statements:

Assume that the pressure in the bubble, at a specific moment, is 30 psi absolute. One can read from Figure 3.6 that water boils at this pressure if the entropy is $\Delta S = 0.368$ BTU/^{o}F lb. From Figure 3.7 it is seen that this entropy increment is given to a particle which has been subjected to a shock front pressure of about 75 kilobars. The boundary of the bubble may be defined to consist of those water particles which are just at the boiling point. The entropy of particles inside this boundary increases with decreasing distance from the center. As one proceeds from the boundary toward the center, one finds particles of water containing small steam bubbles, then particles of wet steam, saturated steam, superheated steam, dissociated steam, and ionized steam, each state merging continuously into the next. In contrast to HE bubbles, neither temperature nor density are uniform throughout a nuclear bubble and there is no clear cut gas-water interface.

Assume that the pressure-distance curve of the shock wave is known and that a peak pressure of 75 kilobars occurs at a distance R = 55 ft. Then, the bubble contains a mass

$$M = \rho \, \frac{4\pi}{3} \, R^3$$
$$= 4.46 \cdot 10^7 \text{ lb}$$
$$= 22.3 \text{ kt.}$$

This mass is in the gaseous, vapor, and liquid states, i.e., the bubble contains water dissociated to various degrees, steam, and "wet" steam.

To summarize: The mass of water vaporized by a nuclear explosion depends on two factors:
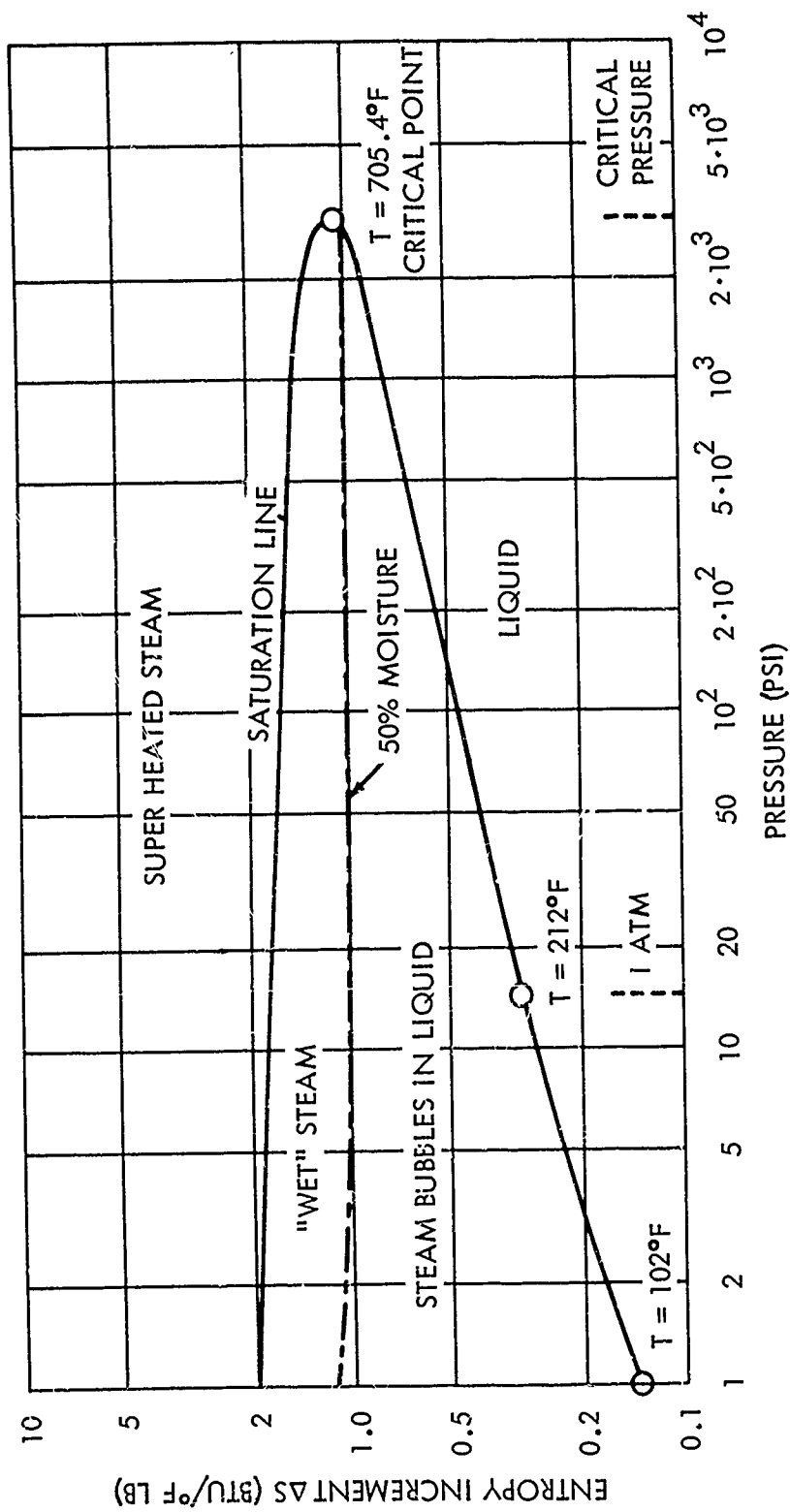
Figure 3.6 — The State of Water as a Function of Entropy and Pressure.

When applied to bubble calculations from nuclear explosions, the abscissa refers to the pressure prevailing in the bubble.

The ordinate is the entropy increment above $32°F$. For shock waves, the entropy commonly refers to the increment above the temperature of the water before the front. A small correction (made in Figure 3.6) is necessary to convert ΔS to the same basis.

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Figure 3.7 – Entropy Increase Incurred by a Particle Over Which a Shock Front of Amplitude p Has Passed.

(a) That the lowest pressure in the bubble is reached at the moment of maximum expansion and is substantially less than the hydrostatic pressure of the water. (The determination of this pressure is an objective of the nuclear bubble theory.)
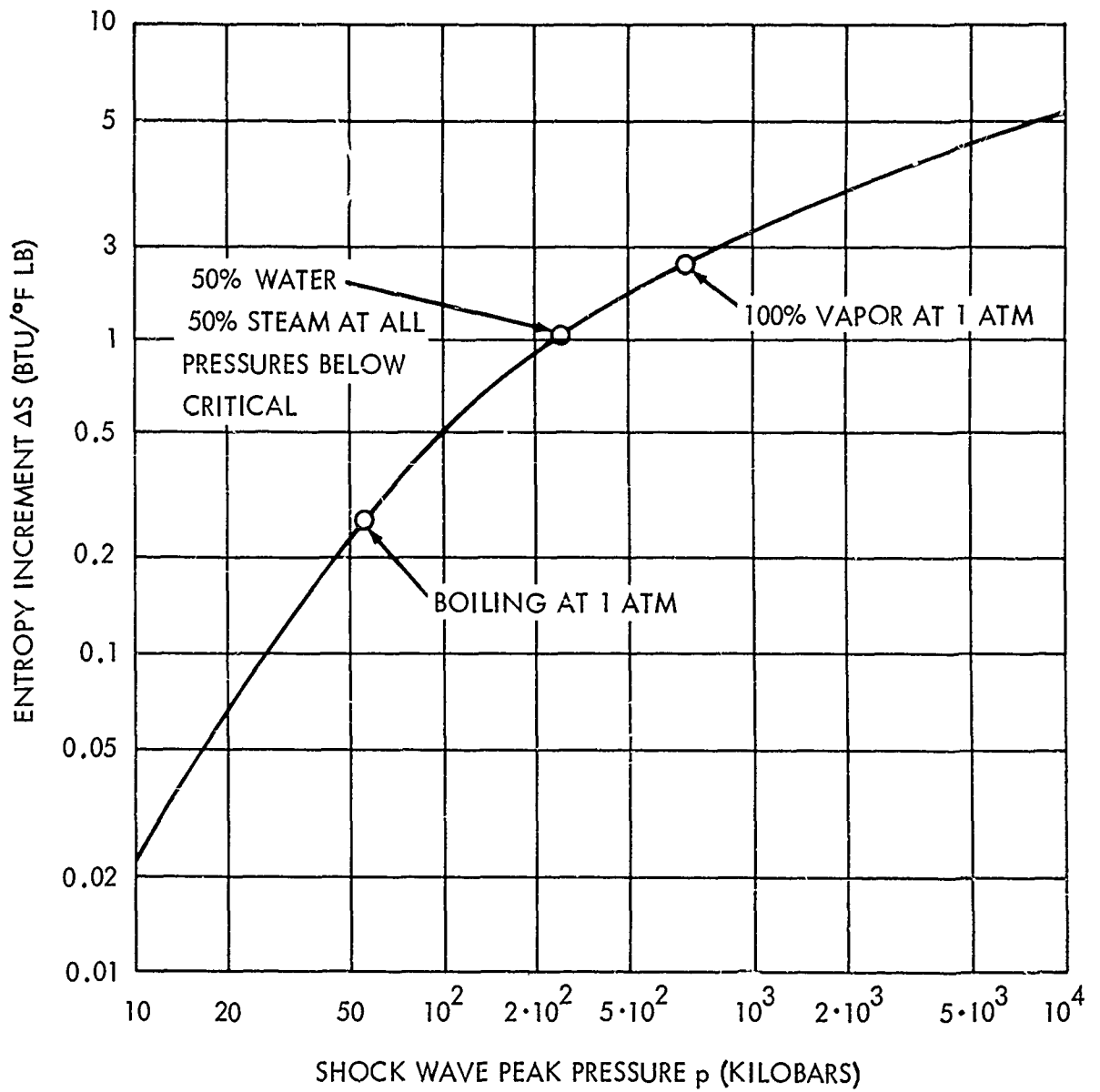
(b) That the water particles suffer an entropy increment at the passage of the shock front.

A water particle will be at the bubble interface when it has been heated by the passage of the shock wave so that it just reaches the boiling point for the pressure prevailing in the bubble.

One result of this situation is that the bubble interface is not fixed and that the mass in the bubble changes. As the bubble expands, water is evaporated; when it contracts, some water is condensed. More details can be found in Chapter VI of this book which deals with bubbles from nuclear explosions.

These discussions show the close interrelationship between the shock front and the nuclear bubble formation: Shock wave and bubble are intimately connected processes.

3.15 EVAPORATION OF WATER IN CONVENTIONAL EXPLOSIONS. A vaporization of water occurs with high explosives. Here, a small portion of the water which was initially near the surface of the charge is vaporized, (Snay, 1957 and 1960). This is obvious from Figure 3.6 which shows that a water particle subjected to a shock front pressure of about 55 kilobars is at the boiling point for 1 atm bubble pressure. HE explosions produce much higher shock pressures. Also, for shallow explosions, the bubble pressure may drop substantially below the atmospheric pressure. Thus, a vaporization similar to that of nuclear bubbles occurs during common explosions, i.e., the water at the bubble interface contains steam bubbles.

Since the shock front pressures of nuclear and conventional explosions are roughly the same at distances which correspond to one HE charge radius,

the amount of water evaporated by a nuclear explosion must be roughly equal to the volume occupied by a conventional charge of equivalent energy.

3.16 SUMMARY. A shock front is a sudden rise in pressure, particle velocity, etc., which propagates with a characteristic velocity through the medium. The propagation velocity is pressure dependent and approaches the speed of sound for low amplitudes. Shock waves are pressure waves which begin with a shock front.

Shock waves are irreversible processes. There is a dissipation of energy as a shock front passes over a particle. This means the temperature of the particle is increased after the disturbance has subsided and the pressure has returned to its original value. The energy dissipation increases with increasing shock wave peak pressure. For vanishing amplitudes, shock waves become reversible processes and can be treated as acoustic waves.

The entropy increase (increment) which a particle undergoes as the shock front passes over it is a convenient tool for the determination of the state of the particle (liquid, vapor, ionized, etc.) when the pressure subsequently declines isentropically. This yields an important key for the understanding of the formation of the steam bubble produced by nuclear explosions.

The amount of water vaporized by a nuclear explosion roughly corresponds to the volume occupied by an HE charge of equal energy.

IV. PROPERTIES OF HIGH AMPLITUDE PRESSURE WAVES

4.1 <u>INTRODUCTION</u>. In the following, a short outline of Riemann's
description of high amplitude pressure waves will be given. It will be noted
that no attempt is made to "solve" the differential equations of the fluid
motion, but that simple elementary transpositions of these equations will
yield the conclusion we need at this point.

The substance of this conclusion will lead to the concept of characteris-
tics. These are a convenient and at the same time far-reaching tool for
treatment of high amplitude pressure waves.

4.2 <u>THE EULER EQUATIONS AND THEIR TRANSFORMATION</u>. It was
mentioned before that considerable energy dissipation occurs within shock
fronts. Outside the region of the steep pressure rise, that is, outside the
front, the dissipation remains small and can be entirely neglected in almost
all studies of explosion pressure waves. (An exception to this is the case of
long distance propagation.)

Therefore, the fluid motion behind the front is sufficiently and accurately
described by the equations of a non-viscous, compressible medium. Follow-
ing Riemann's classic example, we use equations of the Eulerian form. For a
non-viscous medium, the equations for the conservation of momentum and
mass are

$$(4.1) \qquad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0$$

and

$$(4.2) \qquad \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2u\rho}{r} = 0.$$

These equations are supplemented by the requirement of a reversible or
isentropic change of state,

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(4.3)
$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial r} = 0.$$

Remember that

(4.4)
$$\frac{\partial X}{\partial t} + w \frac{\partial X}{\partial r}$$

refers to the rate of change of the magnitude X while the position of X moves with the velocity w. Thus

$$\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}$$

denotes the rate of change of a property of a fluid particle (which by definition moves with the velocity u). For instance, the term $\partial \rho/\partial t + u\partial \rho/\partial r$ which occurs in (4.2) is the rate of change of the density of a fluid particle.

Differential Equation (4.3) states that each particle retains its entropy value, but the possibility exists that adjacent particles will have different entropies. This holds true in the cases which interest us because entropy increases as the shock front passes over a particle. Since this entropy increase depends on amplitude, and since the amplitude decreases as the front proceeds from one particle to the next, particles along a ray from the origin will have slightly different entropies.

In view of (4.3), the term $\partial x/\partial t + u \ \partial x/\partial r$ refers to an isentropic change of the magnitude X. With the sound velocity c, defined by (2.2), we have

(2.2)
$$\left(\frac{\partial p}{\partial \rho} \right)_s = c^2$$

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and by virtue of (4.3)

(4.5) $$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} = c^2 \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right).$$

Equation (4.2) can now be written in the form

(4.2a) $$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \rho c^2 \frac{\partial u}{\partial r} + \frac{2 \rho u c^2}{r} = 0.$$

The combination of (4.1) and 4.2a) yields the following system of equations:

(4.6)
$$\begin{cases} \dfrac{1}{\rho c} \dfrac{\partial p}{\partial t} + \dfrac{u+c}{\rho c} \dfrac{\partial p}{\partial r} + \dfrac{\partial u}{\partial t} + (u+c) \dfrac{\partial u}{\partial r} = -\dfrac{2uc}{r} \\[2ex] \dfrac{1}{\rho c} \dfrac{\partial p}{\partial t} + \dfrac{u-c}{\rho c} \dfrac{\partial p}{\partial r} - \dfrac{\partial u}{\partial t} - (u-c) \dfrac{\partial u}{\partial r} = -\dfrac{2uc}{r}. \end{cases}$$

In contrast to differential equations derived later, these equations are not restricted to a medium of uniform entropy.

4.3 THE RIEMANN FUNCTION. At this point Riemann (1860) achieved a significant simplification of these equations by introducing a new magnitude which today is commonly called the "Riemann Function":

(4.7) $$\sigma = \int_{P_o}^{P} \frac{1}{\rho c} \, dP$$

The integral has to be carried along a path of constant entropy, i.e., the relationship between ρ as well as between c and P to be used in the integration are those of the isentropic (e.g., for an ideal gas $\rho = \rho_o (P/P_o)^{1/\gamma}$). (Note that P designates the absolute pressure, p, the excess pressure above the ambient pressure P_o.)

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Although it has the dimension of a velocity, the Riemann Function σ must not be confused with the particle velocity u. It happens that in special (and important) cases σ and u have the same value, but this is not always true. We must keep in mind that a pressure wave is characterized by three magnitudes: the pressure, the velocity of the medium, and the velocity of propagation. Comparison with the definition (4.7) shows that the Riemann Function is a measure of the pressure of the wave. The fact that this magnitude has the dimension of a velocity should not cause confusion. (Pressure is given in various dimensions, sometimes even in units of a length, e.g., feet of water.) Note also that σ is related, but not identical with the pressure.

With the introduction of the Riemann Function, the equations of motion (4.6) take the form:

$$\frac{\partial}{\partial t}\left\{\sigma + u\right\} + (c + u)\frac{\partial}{\partial r}\left\{\sigma + u\right\} = -\frac{2uc}{r}$$

(4.8)

$$\frac{\partial}{\partial t}\left\{\sigma - u\right\} + (u - c)\frac{\partial}{\partial r}\left\{\sigma - u\right\} = -\frac{2uc}{r} \quad .$$

These equations hold for spherical symmetry and only for an isentropic medium. The case of a plane wave (which was considered by Riemann) is obtained by omitting the right hand side of the equations. These equations permit the following classic interpretation. Application of (4.4) to (4.8) shows that the magnitude $\sigma + u$ propagates with the velocity $c + u$ and the magnitude $\sigma - u$, with $u - c$.

To illustrate this statement, we consider the familiar behavior of sound waves. Here, u is small in comparison with c and is entirely ignored in the propagation term. Further, σ is proportional to the pressure p as shown in (4.9). Then the above statement reads: for a sound wave σ + u propagates with the sonic velocity c, and σ - u, with -c. Obviously, the sign of c indicates the direction of the propagation: the plus sign indicates propagation in the direction of + R, the minus sign in the opposite direction, i.e., towards the origin of the coordinates. The complete solution of the hydrodynamic equations for the acoustic case comprises an outward moving wave (plus sign for u and c) and an inward moving wave (minus sign for u and c). The same holds for high amplitude waves, except that the velocity of propagation is ± (c ± u) where the signs again refer to the direction of the wave motion.

4.4 APPROXIMATIONS. If we set σ = u, which is a good approximation for a water shock wave in the region near the shock front, one finds that the magnitudes σ and u by themselves propagate with c + u. Going a step further, one sometimes finds the statement that the pressure propagates with c ± u, the Rieman Function thereby being replaced by pressure. It must be remembered that such statements are sometimes useful approximations, but that the general case refers to the sum of σ + u or σ - u and not to individual parameters.

A further approximation of rather important nature concerns Riemann's transformation and the use of the magnitude σ. Equations (4.8) hold only for a medium of uniform entropy. As discussed, the medium behind a shock wave of varying amplitude cannot have a uniform entropy since the entropy changes as a function of distance. The introduction of the Riemann Function (4.7) ignores this fact; however, for water the entropy changes are small. Thus, equations (4.8) are useful approximations, particularly for low amplitudes where the changes of entropy become negligibly small.

4.5 <u>SOUND WAVES</u>. For small amplitudes, the particle velocity u in equations (4.8) becomes negligibly small in comparison with c. In this case, $\sigma + u$ propagates with the velocity c. This is why the magnitude c as defined by (4.4) is commonly called "sound velocity," i.e., the propagation velocity of waves of small amplitudes. Strictly speaking, c depends on the temperature (ideal gases) or temperature and pressure (real medium). Therefore, the velocity c varies with the amplitude of the wave, but these variations are small for small amplitudes.

The equations of acoustic waves are obtained from equations (4.8) if u is omitted in the terms (c + u) and (u - c), respectively. Further, c is assumed to be a constant, c_o. If the sound velocity is constant, then the density must be constant, $\rho = \rho_o$. Therefore, the Riemann Function for acoustic waves becomes

$$(4.9) \qquad\qquad \sigma = \frac{p}{\rho_o c_o}$$

where, as before, p designates the excess pressure above the ambient value. This relation again illustrates the nature of the Riemann function as a measure of the pressure amplitude.

Strictly, the acoustic approximation is applicable to the case of infinitely small amplitudes only. However, for water it is a useful approximation up to relatively high pressures, because the low compressibility of water results in a high value of c and in small values of u.

4.6 <u>ACCURACY OF THE ACOUSTIC APPROXIMATION</u>. As an illustration, consider the commonly occurring case where

$$(4.10) \qquad\qquad u = \sigma.$$

Then,

(4.11) $c + u = c_o + \sigma = c_o \ (1 + p/\rho_o \ c_o^2)$,

if c is assumed to be a constant.

Snay et al (1956) give the following interrelationship between sound velocity and pressure which holds for low pressures up to approximately 1 kilobar:

(4.12) $c = c_o \ (1 + \zeta p)$

Then, with the above assumption $u = \sigma$, we obtain from (4.9) and (4.12)

(4.13) $c + u = c_o \ (1 + p \ (\zeta + 1/\rho_o \cdot c_o^2))$

for varying c. Suitable numerical values of the constants are $\rho_o \cdot c_o^2 = 22$ kilobar and $\zeta = 0.108$ kilobar^{-1}. Thus, for a pressure of 1450 psi (0.1 kilobar), we have $c_o + u = 1.0045 \ c_o$, if c is assumed to be a constant, $c + u = 1.0153 \ c_o$, if the change of c with the pressure is accounted for, and $c = 1.0108 \ c_o$. Since $1.0153 \ c_o$ is the approximate value for the propagation velocity, this example shows that the omission of u is less serious than the assumption of a constant sound velocity. The latter error can be reduced if the sound velocity corresponding to a higher pressure than ambient is used for c_o. But, even without this device, the acoustic approximation is useful at much higher pressures than that of our example. Deviations in c + u of 5% to 10% are acceptable, so long as the magnitude of the propagation velocity has no significant bearing on the processes considered. (An example of the latter case is the anomalous surface reflection.) The acoustic approximation is an important tool in underwater explosions research, particularly in studies of underwater explosion damage where the pressures of interest are commonly less than 1 kilobar.

The properties of a spherical acoustic wave can be described by simple equations which are summarized in Appendix A.

For pressures higher than, say, 1 kilobar, the acoustic approximation becomes increasingly inaccurate and fails to describe the behavior of waves of high amplitudes.

4.7 <u>CHARACTERISTICS</u>. The partial differential equations (4.8) allow for simple conclusions concerning the rate of propagation of high amplitude waves. We shall now attempt a further interpretation of these equations. We propose to observe the fluid motion not from a fixed station, but from a moving station. Assume two platforms, one moving with the velocity c + u in the positive r-direction and another moving with u - c in the negative r-direction. If we write the hydrodynamic equations for these two moving points of observation, the partial differential equations (4.8) are reduced to ordinary ones. This follows immediately from an application of equation (4.4). We obtain

$$(4.14) \qquad \frac{1}{\rho c}\frac{dp}{dt} + \frac{du}{dt} + \frac{2uc}{r} = 0$$

applicable to a point which moves with the velocity c + u (in the direction of r). Similarly, the differential equation

$$(4.15) \qquad \frac{1}{\rho c}\frac{dp}{dt} - \frac{du}{dt} + \frac{2uc}{r} = 0$$

holds for a point moving with the velocity u - c in the direction of -r.

Consider the travel of two such platforms. We may plot their paths in an r-t diagram. Then, two curves are obtained which at every point have the inclinations

$$(4.16) \qquad \frac{dr}{dt} = c + u$$

and

$$(4.17) \qquad \frac{dr}{dt} = u - c,$$

respectively. Such curves are called characteristics* and we restate: the ordinary differential equation (4.14) holds along a (c + u)-characteristic , and (4.15) along a (u - c)-characteristic. The (c ⊢ u)-characteristic describes a wave which propagates in the positive r-direction, the other, a wave going in the negative r-direction.

Thus, we have reduced the system of partial differential equations (4.8) into a system of ordinary differential equations. If an immediate construction of the characteristics were possible, a relatively simple numerical integration of these ordinary differential equations would yield solutions for any given initial condition without further complication. Since c and u depend on the solution of equation (4.14) and equation (4.15), such a construction is, in general, not possible. However, special cases are tractable. It is also possible to obtain numerical solutions by means of step by step calculations, where only a small portion of the total field of the characteristics is considered at a time (see Article 4.12).

4.8 SIMPLE PRESSURE WAVES. The concept of the characteristic affords a convenient discussion of the behavior of high-amplitude waves. For expediency, we begin this discussion with the so-called "simple waves."

Consider plane pressure waves without shock fronts which propagate through a homogeneous and isentropic medium. Then, the introduction of the Riemann Function is permissible and we obtain from equations (4.8)

(4.18a) $\dfrac{d}{dt} (\sigma + u) = 0$ along the (c + u)-characteristic

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* In the mathematical theory of partial differential equations, characteristics have a much farther-reaching significance than the kinematic interpretation given here, e.g., see Courant and Friedrichs (1948), pages 40 to 68. For the purpose of our study, it is not necessary to go into those more complicated details.

and

(4.18b) $\dfrac{d}{dt}$ $(\sigma - u) = 0$ along the $(u - c)$-characteristic;

i.e., $\sigma + u$ or $\sigma - u$ is constant along the respective characteristic. This
further illustrates the physical meaning of the characteristics. They describe
the propagation of a "point of the wave."

In the case of an ocean wave, one would call either the crest or the
trough, or any other specific location, a "point of the wave," implying that,
as time goes on, such a point will always remain the crest, or the trough,
etc.— In an entirely analogous way, a point of a plane pressure wave is de-
fined by a specific value of $\sigma + u$, or $\sigma - u$, respectively. These are magni-
tudes which do not change as the wave moves along, in the same way the
crest of an ocean wave remains the crest. Thus, the characteristic is the
time-distance location of a point of the wave. (For spherical waves, $\sigma + u$ and
$\sigma - u$ change with distance. The concept of the characteristic provides the
general although not so obvious definition of the point of the wave.) Since
each characteristic describes the motion of one point of a wave, a family of
characteristics is needed to describe the propagation of a pressure pulse.

The general plane isentropic wave, i.e., a wave having any given
value for σ and u, is obtained by superposition of two waves. One of these
moves in the positive r-direction. At each point the sum of σ and u remains
a constant, while the other wave moves in the negative r-direction and $\sigma - u$
is constant.

The magnitudes, $\sigma + u$ and $\sigma - u$, are sometimes called "Riemann
invariants." This term is appropriate for isentropic plane waves only. In all
other cases these magnitudes are not constants.

"Simple" waves are waves for which either $\sigma = u$ or $\sigma = - u$. Since in
the isentropic case c can be expressed as a function of σ alone and since σ
and u are constants along the characteristics, c and u also must be constants:
the characteristics of simple waves are straight lines and, in this case, can

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be immediately constructed. This has been done in Figure 4.1.

If $\sigma = u$, the amplitude propagated along the $(u - c)$-characteristic is zero and it is not necessary to consider the $(u - c)$-characteristic, but only the $(c + u)$-characteristic. Assume that the amplitudes, namely σ and u, are given as a function of time at the distance $r = r_o$ and that we wish to study the propagation of this pulse. This initial condition is shown in the amplitude-time diagram of Figure 4.1 where σ as well as u are plotted versus t, (the time is given in multiples of r_o/c_o, the distance in multiples of r_o). Since $\sigma = u$, only one curve is shown and is labeled r_o.

To construct the characteristics, a relation for c is needed. For the example of Figure 4.1, the simple equation

$$(4.19) \qquad\qquad c = c_o + 3\sigma$$

has been used which gives acceptable accuracy for water in the pressure range between 10 and 50 kilobars. The value c_o is the sound velocity for $p = 0$. By means of equation (4.19), the sound velocity can be determined for the initially given data. Thus, one must visualize that in the $r - t$ plane the initial condition of the wave, namely, values of σ, u, and c, are given along the parallel to the t-axis, $r = r_o$.

Figure 4.1 shows the characteristics emerging from this parallel $r = r_o$. Their inclination is $\tan \alpha = c + u = c_o + 4\sigma$. Since zero amplitude is assumed for the head as well as the tail end of the pulse, the characteristics corresponding to these points are parallels. (Note that the head is on the left side in the σ, t-diagram as is commonly the case in pressure-time diagrams. The head passes the point of observation first (i.e., for small values of t), the tail last (i.e., for large values of t). If a pressure-distance or σ-distance curve is considered for a constant moment of time, the head is at the right.)

In the specific case of our example, σ and u are constant along each characteristic. Hence, such characteristic diagrams may be visualized as
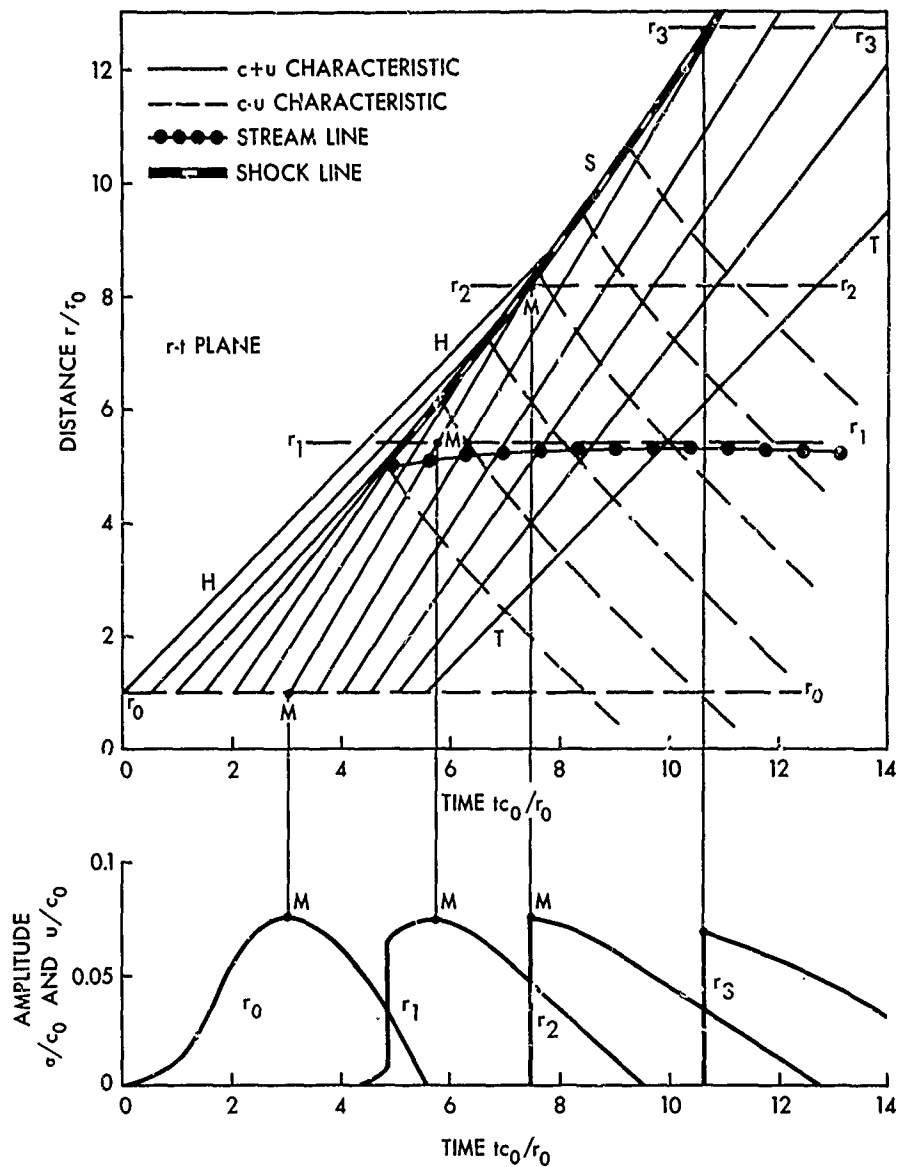
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Figure 4.1 - Characteristic Diagram of the Propagation of a Plane Wave

The upper diagram shows the characteristics in the r-t plane. Each of these curves gives the time-distance location of a specific property moving with the wave. For instance, the characteristic M-M-M gives the path of the maximum amplitude, H-H, that of the head, and T-T, that of the tail of the pressure pulse. The broken curve S gives the path of the shock-front.

The lower diagram shows the corresponding amplitudes of the wave as a function of time for four distances, r_o through r_3, marked in the r-t plane above. The graph illustrates the change of the pulse shape and the formation of the shock-front.

Reduced magnitudes are used throughout. The amplitudes σ and u are given in multiples of c_o, the distance in multiples of r_o, the time in multiples of r_o/c_o. For simplicity it is assumed that $\sigma = u$.

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contour maps similar to topographic (geodetic) maps where each contour line designates the elevation. In our diagram, the contour lines are the characteristics and each designates constant values of the amplitudes σ and u.

The higher the amplitude, the steeper the inclination of the characteristic. The consequence of this is typical of all compression waves: some of the characteristics converge and intersect; others diverge. This produces a change of shape of the pressure pulse, namely, a steepening of the pulse for convergent characteristics. Divergence of the characteristics results in a flattening of the pulse shape. Intersection of two characteristics corresponds to a shock front exactly as the intersection of two topographic contour lines indicates a precipice.

4.9 FORMATION OF SHOCK FRONTS. The characteristics must terminate at the shock line as shown in Figure 4.1, because the equations for non-viscous fluids do not hold for the shock front. Continuation of the characteristics beyond their intersection would lead to an overhanging face, i.e., to the physical impossibility of having three values of σ and u at the same time and the same space (shown in Figure 3.2).

As discussed, the Rankine-Hugoniot relations must be used to describe the properties of the shock front. The inclination of the shock curve in the r, t-diagram is

(4.20)
$$\left(\frac{dr}{dt}\right)_{Sh} = U,$$

where U is the propagation velocity of the shock front. In order for a shock front to persist, U must be smaller than c + u. This is apparent from Figure 4.1: the shock line (loosely called U-characteristic) and the (c + u)-characteristic must intersect, otherwise the shock line cannot form the envelope of the intersections of the individual (c + u)-characteristics. For water, U is always smaller than c + u. The expression for U, for instance, which

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corresponds to $c = c_o + 3\sigma$, is $U = c_o + 2\sigma$, therefore $c + u = c_o + 3\sigma + u$ is larger than $U = c_o + 2\sigma$. In such a case, one speaks of a "stable" shock front.

If the shock front relations do not result in $\sigma = u$, a "reflection at the shock front" takes place. This amounts to the generation of a new wave which originates at the shock front and runs backward into the oncoming original wave. Such a wave is described by the $(u - c)$-characteristics along which $\sigma - u$ is not zero, but finite. The superposition of these two waves at a point on the shock front must satisfy the following condition:

$$\underbrace{\sigma_a + u_a}_{\text{oncoming wave}} + \underbrace{(\sigma_b - u_b)}_{\text{reflected wave}} = \underbrace{\sigma_s + u_s}_{\substack{\text{As required by} \\ \text{Rankine-Hugoniot} \\ \text{condition}}} .$$

This reflected wave affects the entire pressure pulse and may considerably change the shape and amplitude of the pulse if it has a high amplitude. The characteristics cease to be straight lines where intersections between $(c + u)$- and $(u - c)$-characteristics occur and neither σ nor u is constant along them. (In Figure 4.1 the assumption has been made that σ_b and u_b are small.) If the Rankine-Hugoniot relations yield $\sigma_s = u_s$, such reflection at the shock front does not take place. This condition is approximately satisfied for shock waves in water.

In the region "behind the front," the assumption of constant entropy and the use of the Riemann Function σ must be dropped. More precisely, "behind the shock front" refers to fluid particles over which a shock front has passed. The path of such particles, the streamline, is given in the r-t plane by a curve which has the same inclination at all points

(4.21) $$\frac{dr}{dt} = u.$$

It is sometimes loosely called the u-characteristic. In accordance with equation (4.3), these are lines of constant entropy. In Figure 4.1, only the line which passes through the beginning of the shock line is shown. The region below this curve is isentropic. Above this boundary line, c is a function of two magnitudes, c_o and σ, for example. It is obvious that the problem is now considerably complicated. However, our conclusions on the change of pulse shape, the formation of a shock front, lines of constant entropy, etc., remain valid, although details cannot be as simple as those for simple waves.

All characteristics are parallel for acoustic waves. Acoustic waves do not change their shape and cannot form shock fronts, but shock waves can be represented by the acoustic approximation. (See Appendix A.)

4.10 FURTHER DETAILS ON THE FORMATION OF SHOCK FRONTS. Figure 4.1 illustrated three further conclusions:

(a) Rarefaction waves have divergent characteristics and do not form shock fronts. In our example, the characteristic M-M refers to the maximum of the pressure pulse. The portion of the pulse behind point M in the lower diagram of Figure 4.1 is a rarefaction wave. The characteristics in the upper diagram are divergent at the right-hand side of M-M. They cannot intersect each other as time increases and cannot form a shock front, but they can intersect with an existing shock line, as seen in Figure 4.1 where the next characteristic at the right of M-M approaches and intersects the shock line.

(b) Shock fronts, after being formed, can be and are commonly followed by a rarefaction wave. The lower portion of Figure 4.1 shows the development of the σ history (which roughly corresponds to the pressure history) for four distances r_o, r_1, r_2, and r_3.

These four curves demonstrate the steepening of the compressive portion of the pressure wave and flattening of the rarefactive portion. At the distance

r_1 the shock front has just been formed and is preceded by a small precursor wave. The pressure behind the shock front continues to rise and we now have the case where the front is followed by a compression. The shock front recedes rapidly with respect to the remainder of the wave (because U<c + u) and at $r = r_2$ the front coincides with the maximum, M. From this point on, a rarefaction immediately follows the front. Our example (which is not exaggerated) shows how quickly the portion of the pressure wave which precedes the maximum of the pulse disappears so that subsequently the maximum occurs at the front.

(c) Once the maximum of a plane pressure wave is at the front, the maximum amplitude will decrease with distance (compare the curve labeled r_3 in Figure 4.1). This behavior of a plane, simple shock wave is in contrast to that of a plane, simple pressure wave whose amplitude remains constant.

This decrease of the peak amplitude is readily explained by Figure 4.1. The characteristics of the rarefaction phase of the wave intersect one after the other with the shock line (only the intersection of the first characteristic, specifically that at the right of x-x, is shown. The other intersections are outside the field of Figure 4.1). If a reflection at the shock front does not take place, the front assumes exactly the amplitude of the pertinent c + u characteristic at the point of intersection. So long as each of the successive characteristics "carries" a lower amplitude than the preceding one, the shock front peak pressure must decrease.

If the $\sigma - t$ curve, labeled r_2, is compared with the curve labeled r_0, it will be seen that the portion ahead of the point x has vanished. The missing portion corresponds to the energy dissipated in the shock front.

4.11 SPHERICAL PRESSURE WAVES, AFTERFLOW, AND UNDERWATER EXPLOSION BUBBLES. Equations (4.8) describe the case of an isentropic spherical pressure wave. They can be interpreted as

(4.22a) $$\frac{d}{dt}(\sigma + u) = -\frac{2uc}{r}$$ along (c + u)-characteristics

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and

(4.22b) $\quad \dfrac{d}{dt}(\sigma - u) \approx -\dfrac{2uc}{r}\quad$ along (u - c)-characteristics

Thus, $(\sigma + u)$ or $(\sigma - u)$ are not constant along the characteristics as in the plane case, but change according to the differential equations. This formulation illuminates the well known situation that the amplitude of spherical waves decreases with increasing distance, whereas that of plane isentropic waves remains constant. As a consequence, the characteristics are not straight lines for spherical high amplitude waves.

Equations (4.22) have another significant consequence. Even if σ and u are initially equal, a difference, $(\sigma - u)$, builds up as a shperical wave moves ahead. Thus, both equations (4.22a) and (4.22b) are needed to determine σ and u, or in other words, the (c + u)- as well as the (u - c)- characteristics must be used in the case of spherical waves.

This immediately raises the question: Is it possible to have a spherical pressure wave which moves in one direction only, or does a spherical wave generate a wave in the opposite direction, as occurs in the case of the reflection at the shock front, Figure 4.1? The answer is that spherical waves running in one direction are possible despite the need for both systems of characteristics. But, such waves must have certain properties.

Consider a solitary pulse, i.e., a pulse where the amplitude is zero at the head of the wave as well as at its tail end. Thus, $\sigma = 0$ and u = 0 at these points. This is possible only if the integral of the right-hand side of equation (4.22b) vanishes. Then, for an outward going solitary wave,

(4.23) $\qquad \displaystyle\int_{t_H}^{t_T} \dfrac{uc}{r}\, dt = 0,$

where this integral must be carried along a (u - c)-characteristic. The limits, t_H and t_T, refer to the time where head or tail, respectively, arrive at the

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(u - c)-characteristic. Figure 4.2 illustrates the characteristics as well as the path of this integration. Excluding the trivial case u = 0 (i.e., no wave at all), the integral (4.23) will vanish only if the integrand assumes positive as well as negative values. Since c cannot be negative, u and σ must change signs. This means: A solitary spherical pressure pulse must involve positive as well as negative amplitudes.

This result is of considerable importance for underwater explosions since it indicates the existence of the pulsating gas or steam bubble.

On the basis of physical considerations, it would be difficult to visualize that the pressure pulses emitted from explosions would also produce backward running waves (reflections at the shock front excluded). Pressure waves from explosions indeed resemble solitary waves, i.e., the particle velocity oscillates between positive and negative values. These oscillations, which are noticeable at large distances, are manifestations of the oscillating bubble.

Before going into the discussion of the interrelation with the bubble, it is necessary to stress a semantic implication: what is commonly called the underwater explosion shock wave, comprises only one portion and not the total of the wave emitted by the explosion. It refers to a pressure wave which decreases with distance but is otherwise treated as a plane wave, namely, it is assumed that σ always equals u. (The fact that shock waves are not isentropic processes is also ignored.) Since σ - u is essentially zero at the shock front, and since the (σ - u)-term builds up slowly behind the shock front, this treatment is appropriate for the first portion of the shock wave. The term ignored thereby, namely (u - σ), is called the afterflow. Hence:

> Total underwater explosion pressure wave = "Shock Wave" + "Afterflow."
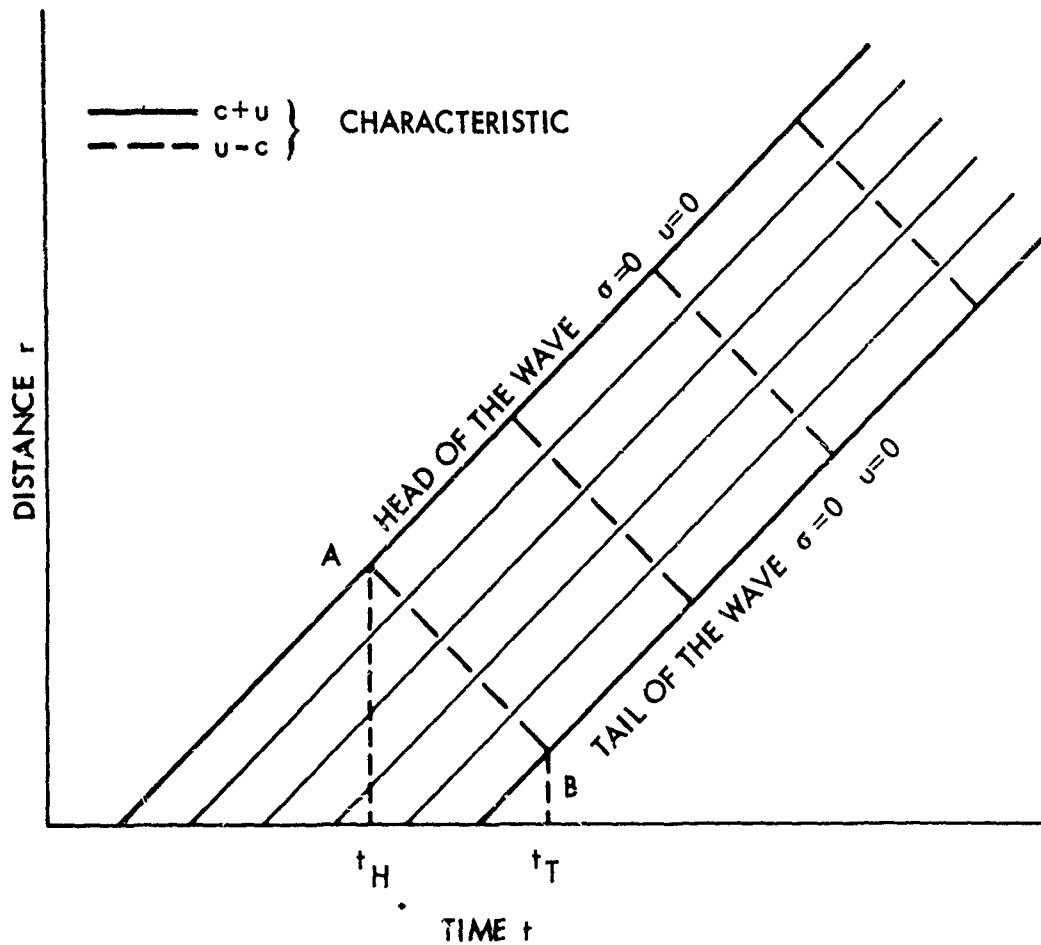
Figure 4.2 - Characteristics of a Solitary Pulse.

(Low amplitudes are assumed in drawing this graph.)

The integral $\int (uc/r)dt$ must be evaluated along (u-c) characteristics such as AB and must vanish for solitary pulses.

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Figure 4.3 shows the complete history of pressure and particle velocity
for a spherical shock wave. All magnitudes are given in dimensionless form.
The peak pressure at the shock front and the peak velocity are used to obtain
the reduced pressure $\tilde{p} = p/p_m$ and reduced velocity $\tilde{u} = u/u_m$. The time
constant θ of the shock wave is used for the reduced time $\tilde{t}$. The figure
shows that initially $\tilde{\sigma} = \tilde{u}$ and that the reduced afterflow $\tilde{u} - \tilde{\sigma}$ is built up
slowly. The values $\tilde{p}$, $\tilde{\sigma}$ and $\tilde{u}$ change sign as is necessary for a spherical
wave running in one direction. After a prolonged period of negative pressure,
a secondary pressure pulse appears. Pressure and velocity will continue to
oscillate in this fashion.

These pressure oscillations correspond to the bubble pulsations. The
bubble maximum will occur at the point where $\tilde{u}$ becomes negative and $\tilde{p}$
reaches a minimum. The second positive pressure pulse is called the bubble
pulse. It is emitted when the bubble has contracted to its minimum size.

For plane waves σ and u coincide, and there is no afterflow; therefore,
plane pressure waves cannot produce a pulsating bubble.

The afterflow decreases with the square of the distance from the origin
(see Appendix A). This means the afterflow has a pronounced effect at small
distances. The motion of the bubble corresponds exactly to the afterflow
evaluated for the particle which forms the bubble interface. Strict calcula-
tions are difficult; however, useful approximations can be made on the basis
that the afterflow corresponds roughly to an incompressible fluid motion, as
further discussed in Appendix A.

The definition of the shock wave given above needs further development.
If we consider the pressure history by itself, the question arises as to which
portion of the pressure record should be attributed to the bubble and which to
the shock wave. The second pressure pulse shown in Figure 4.3 as well as
the underpressure phase are clearly "bubble phenomena." When retracing the
pressure record toward the shock front, a term will appear which is not
connected with the bubble. One might be inclined to call this portion "the
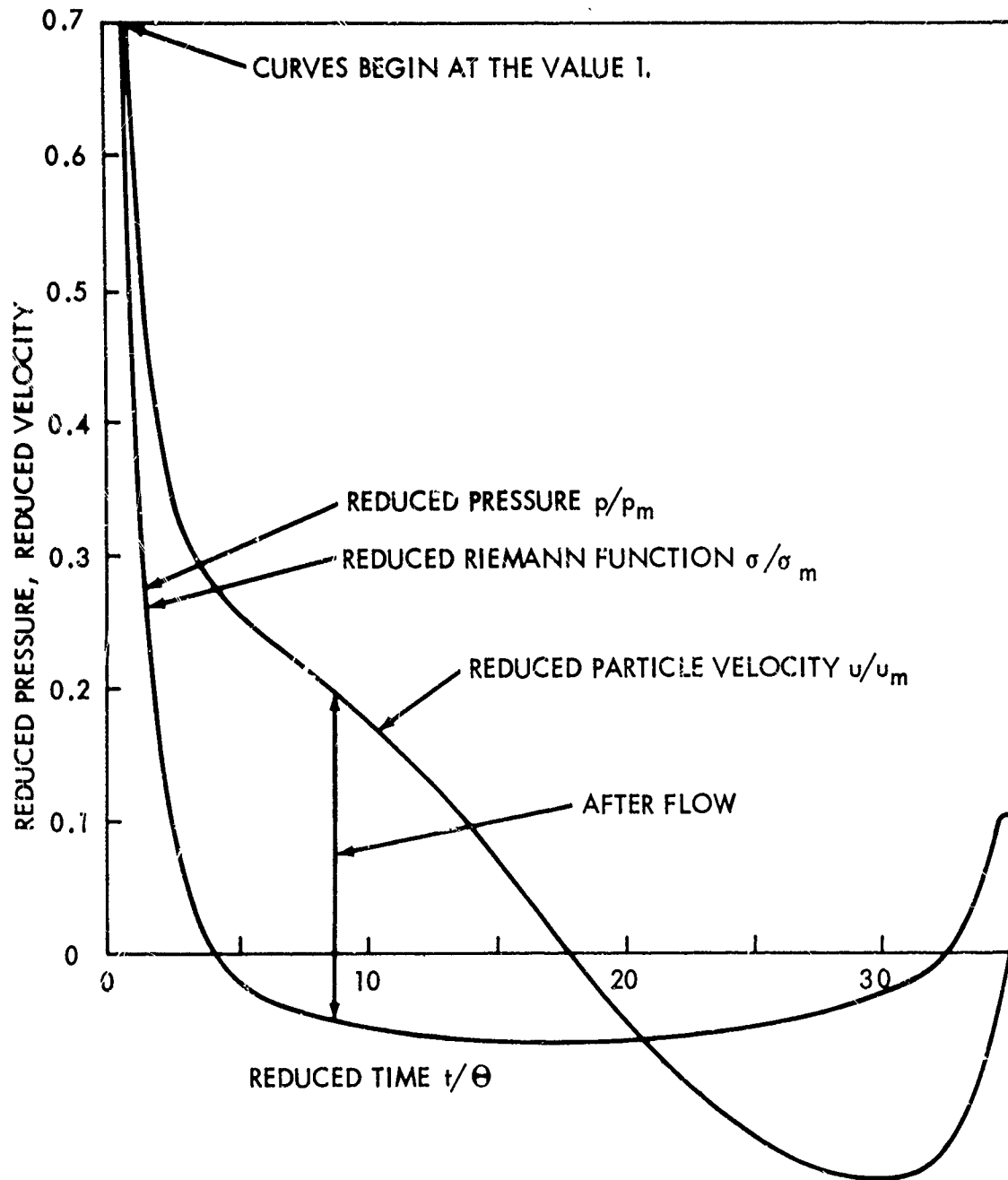
(Text continued on page 67).

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Figure 4.3 - Pressure (or Riemann Function)
and Particle Velocity as Functions of Time

(Schematic Illustration. Details depend on Depth of Explosion and Distance from Charge.)

stretches of the characteristics. In this way, the network of characteristics and the solution along them can be calculated step by step in an analogous way to the simple case of Figure 4.1.

The introduction of electronic computers gave a great impetus to such calculations. In fact, only those computers have made such calculations feasible, although valiant efforts had been previously made using desk computers and graphical methods (Pfeiffer and Meier Koenig, 1943).

Even though this "method of characteristics" is suitable for calculations of compressible fluid motions and constitutes the classic approach, it has not found widespread use in the field of explosions. This is because the method of von Neumann-Richtmyer (1950), which accomplishes a direct numerical step by step integration of the partial differential equations, is simpler in many respects. One of its advantages is that no special provisions are necessary for shock fronts. In the method of characteristics, a strict distinction between the $c \pm u$ characteristics and the U (or shock) characteristic must be made which is cumbersome, yet yields sharp shock fronts and permits the use of the equation of the non-viscous fluid motion behind the front. The von Neumann- Richtmyer method uses Lagrange-type equations which include a viscosity term. This makes it possible to integrate over the shock front. Because of the finite mesh size, the shock front does not appear sharp but as a rather gradual rise with the width of the front unrealistically wide. The shock wave has a rounded maximum which is often somewhat lower than the actual peak. Also, under some conditions the region behind the front is adversely affected. These disadvantages have been partly reduced by an improved version of the viscosity term which is not rigorous for the hydrodynamic partial differential equations, but is tailored for the finite difference method in such a way that the viscous effect is greatly enhanced for the shock region. Still, estimates and experience are necessary to locate the position of the front and to determine the shock wave peak pressure.

These peculiarities of the von Neumann- Richtmyer method by no means detract from its immense value. It is widely and successfully used in the field of explosions. Improvements are constantly being made such as the "particle in cell" or "energy in cell" methods. Shortcomings are being eliminated.

These powerful tools have not found widespread use in the field of nuclear underwater explosions. For this reason, an explicit description of these methods is omitted in this paper. There is no doubt that in the near future important results will be obtained with these methods. Therefore, it is planned to devote a full section to this subject in the second edition of this paper.

4.13 SUMMARY. High amplitude pressure waves propagate with the velocity $c + u$ in the positive r-direction and, with $u-c$ in the negative r-direction. The pulse shape of such waves changes as they propagate through the medium. If the initial rise of the pressure at the head of the wave is gradual, the steepness of this rise will increase until a shock front is formed.

For acoustic pressure waves, i.e., waves of small amplitude, the particle velocity u is negligibly small compared with the sound velocity c. For small amplitudes, c is a constant. Such waves propagate with a constant velocity c, the sound velocity. They do not change their shape and cannot form a shock front.

For underwater explosions, the acoustic approximation often gives a sufficiently accurate description of the flow process and is an indispensable tool for the solution of interaction problems.

Outward moving spherical pressure waves leave a mass flow behind, the afterflow. This flow leads to the formation of a cavity, namely, the underwater explosion bubble in the case of common explosions, a steam bubble for nuclear explosions, and a cavitation bubble in the case of gasless explosions or strong solitary waves produced by suitable transducers.

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APPENDIX A

ACOUSTIC WAVES

A1.1 SIGNIFICANCE OF THE ACOUSTIC APPROXIMATION. In Article 4.6 it was shown that the accuracy of the acoustic approximation remains acceptable for an underwater pressure wave if the amplitude does not exceed pressures of 1 kilobar, for example. An additional restriction is that only propagation over relatively short ranges be considered. For long range propagation of explosion waves, high amplitude effects cannot be omitted, even at very low pressures. An example is the anomalous surface reflection, Rosenbaum-Snay (1956), where minute high-amplitude (non-linear) effects build up and ultimately produce a drastic deviation from the result of the acoustic theory.

The acoustic theory is a linear theory and affords analytical solutions which often are of considerable complexity but are tractable. For high amplitude waves the complexities are almost insurmountable even when high-speed electronic computers are used. Examples are: reflections of shock waves from elastic or porous bottoms or their refraction by the inhomogeneity of the ocean; cavitation processes; interactions with yielding structures; etc. In these cases, the acoustic approximation is an indispensable tool.

A2.2 VELOCITY POTENTIAL AND WAVE EQUATION. As discussed in Article 4.5, the acoustic approximation can be obtained if the particle velocity u is ignored in comparison with the sound velocity c, i.e., if the Mach number is zero. The hydrodynamic equations (4.8) have a solution in this case which will be shown presently.

A fluid motion of this type is irrotational, as discussed in Article 2.7, and such motions can be described by a velocity potential φ which is defined by

(A1.1) $$- \nabla \varphi = \vec{u},$$

where $\vec{u}$ is the velocity vector, φ is a scalar and equation (A1.1) states that the gradient of the velocity potential is the velocity vector.

For the case of a spherical fluid motion considered in this paper, equation (A1.1) becomes simply

(A1.2) $$- \frac{\partial \varphi}{\partial r} = u.$$

If the velocity potential is introduced into the properly simplified hydro-dynamic equations (e.g. (4.8) with u omitted in the propagation term $\pm (c \pm u)$ and c set constant $= c_o$) one obtains the wave equation,

(A1.3) $$\nabla^2 \varphi = \frac{1}{c_o^2} \frac{\partial^2 \varphi}{\partial t^2}.$$

The subscript o designates the constancy of c. For simplicity, this subscript will be omitted. For a spherical wave, equation (A1.3) has the general solution,

(A1.4) $$\varphi = \frac{1}{r} f(t-r/c) + \frac{1}{r} g(t+r/c),$$

which represents the inward and outward running waves discussed in Article 4.3. f and g are arbitrary functions of the arguments[*] (t-r/c) and (t+r/c). These arguments are often called the "retarded times." Obviously, r/c is the time required for the wave to travel from the origin of the coordinates to the distance r, while t-r/c marks the time when the head of the wave or any other characteristic point of the wave leaves the origin. (In the language of

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[*] Independent variables.

explosion research, it is said that t counts time from the moment of explosion
and r measures the distance from the center of the explosion.)

The function f has always the same value at an equal retarded time.
(The analogous statement can be made for g.) This corresponds to the moving
platform of Article 4.7 from which a "point of the wave" is observed. In our
case, f and g retain their value when seen from these points of observation.
The equations t-r/c = const. and t+r/c = const. are equations of the
characteristics. As discussed in Article 4.7, they are straight lines for
acoustic waves.

The particle velocity of a spherical acoustic wave follows immediately
from (A1.2), viz.,

$$(A1.5) \qquad u = \frac{1}{r} f'(t-r/c) + \frac{1}{r^2} f(t-r/c)$$

$$+ \frac{1}{r} g'(t+r/c) + \frac{1}{r^2} g(t+r/c),$$

where the prime designates the derivative of f or g with respect to its
argument. It is seen that the particle velocity of each wave consists of
two terms, f' and f or g' and g, where f and g represent the afterflow
discussed in Article 4.11.

The pressure of an acoustic wave is given by

$$(A1.6) \qquad p = \rho \frac{\partial \varphi}{\partial t} .$$

This relation stems from the "Bernoulli Equation" which will be described
below.

With (A1.4) we obtain for the pressure

$$(A1.7) \qquad p = \rho c\, f'(t-r/c) - \rho c\, g'(t+r/c).$$

(Note that g' must be negative for a compression wave. This yields a nega-
tive particle velocity for the inward moving wave in accordance with the

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remarks made in Article 4.3.) The density ρ and the sound velocity c are constants in the acoustic theory. Their product ρc is called the acoustic impedance. For water, ρc changes slightly with the temperature T:

$$\rho c = 5.14 + 0.0144 \ T(^\circ C) \text{ for fresh water or}$$

$$\rho c = 5.58 + 0.0065 \ T(^\circ C) \text{ for sea water.}$$

The units of ρc given above are in psi·sec/inch.

Remembering the definition of the Riemann Function, (4.7) or (4.9), we can write

$$u = \sigma + \frac{1}{r^2} f \cdot \frac{1}{r^2} g \quad \text{and}$$

(A1.8)

$$\sigma = \frac{1}{r} f' - \frac{1}{r} g' \ .$$

This relation has been used to construct Figure 4.3.

For an outward moving wave, the following formulation is useful:

$$p(r,t) = \frac{1}{r} f_1 (t-c/r) \quad \text{and}$$

(A1.9)

$$u(r,t) = \frac{p(r,t)}{\rho c} - \frac{1}{r\rho} \int_{t_o}^{t} p(r,t)dt.$$

Here, the lower limit t_o refers to the moment where $\sigma = u$.

A1.3 UNDERWATER EXPLOSION WAVES AND BUBBLES. An approximate expression for an underwater explosion shock wave is

$$p = \frac{p_1 a_1}{r} e^{-(t-r/c)/\theta} \quad \text{and}$$

(A1.10)

$$u = \frac{u_1 a_1}{r} \left[e^{-(t-r/c)/\theta} + \frac{c\theta}{r} \left\{ 1 - e^{-(t-r/c)/\theta} \right\} \right]$$

with

$$u_1 = p_1/\rho c.$$

Here, p_1 is the shock wave peak pressure at the distance $r = a_1$ and θ is the time constant of the wave.

The "time constant" of an acoustic wave is a true constant, whereas the θ of a high-amplitude wave changes with distance. This reflects the statements of Article 4.8 that pressure waves change their shape as they propagate, but that acoustic waves retain their shape.

According to the above equations, the pressure of an acoustic wave decreases inversely proportionally with distance. No energy dissipation occurs with this mode of propagation. Underwater explosion shock waves dissipate energy and, therefore, decay more strongly with distance. The peak pressure and time constant are functions of the charge weight W (in lb.) and distance R (in ft.). For TNT they read as follows:

$$p_{max} = 21,600 \left(\frac{W^{\frac{1}{3}}}{R} \right)^{1.13} \quad \text{psi}$$

(A1.11)

$$\theta = 0.056 \, W^{\frac{1}{3}} \left(\frac{W^{\frac{1}{3}}}{R} \right)^{-0.22} \quad \text{millisec.}$$

The acoustic equations (A1.10) are particularly useful and accurate, if applied to distances that are not very different from a_1 and if p_1 and θ are calculated for $R = a_1$ using (A1.11). If $c = c(p_1)$ is introduced, an exact linearization of the problem is obtained (compare with Article 4.6). This method, however, is not the only one which allows for an adaptation of the acoustic approximation to explosion shock waves. These methods depend on the nature of the problem and their discussion goes beyond the scope of this outline. Many utilize the fact that the greatest difference between these two types of waves occur near the shock front and that the remainder of the wave behaves essentially like an acoustic wave.

Returning to equation (A1.10), we note that the term in the brackets represents the afterflow. For $t \gg \theta$, we obtain for the particle velocity

$$(A1.12) \qquad u \to \frac{u_1 \, c \, \theta \, a_1}{r^2} \qquad \text{for } t \gg \theta$$

which is equivalent to

$$(A1.12a) \qquad u \to \frac{a \cdot a^2}{r^2} \, ,$$

where $a\cdot = u_1 \, c \, \theta / a$ is the velocity at $r = a$.

This expression is interesting in two respects. We may interpret a as the radius of the explosion bubble (of either a nuclear or chemical explosion). Since θ is very small in comparison with typical times of the bubble motion (the period of the bubble pulsation may be 100 times larger than θ), the assumption $t \gg \theta$ is appropriate. Further, the pressures connected with the bubble motion are low which, to some extent, justifies the acoustic treatment (see, however, Article A1.5).

Expression (A1.12a) leads to the following conclusions:

(a) This relation is exactly that of the incompressible fluid motion. Hence, the afterflow resembles the motion of an incompressible fluid, although this term has been obtained from the equations of a compression wave where compressibility is the predominant factor. Since the afterflow is the important process that causes the bubble expansion and pulsation, the use of the equations of the incompressible fluid motion in the bubble theory appears to be justified. The classic bubble theory indeed uses this fluid model with success. However, the afterflow is only a part of the total phenomenon. The classic bubble theory, therefore, is an approximation and, consequently, special methods are needed to make it useful for practical applications.

(b) The velocity $a\cdot$ in (A1.12a) is constant. Thus, the bubble caused by the pressure wave equation (A1.10) keeps on expanding indefinitely.

We conclude that equation (A1.10) is an unrealistic description of a solitary pressure wave. As described in Article 4.11, a solitary pulse must include positive as well as negative pressure phases. The strict conditions are given there. For an acoustic wave, it appears from equation (A1.10) that the total impulse, i.e., the pressure-time integral extended to t→∞, must vanish.

Although the equation (A1.10) for the "exponential pressure wave" is, strictly speaking, incorrect if used for long durations, it is very useful to describe the first part of the shock wave.

A1.4 THE ACOUSTIC APPROXIMATION OF A SHOCK FRONT. Frequently, the question is raised: Can a shock wave be represented by an acoustic wave? Equations (A1.10) do that. The statements

$$p = 0 \qquad \text{for } t < r/c$$

(A1.13)
$$\left.
\begin{aligned}
p &= \frac{p_1 \, a_1}{r} \, e^{-(t-r/c)/\theta} \\[2mm]
u &= p/\rho c + \text{afterflow}
\end{aligned}
\right\} \quad \text{for } t \geq r/c$$

describe a discontinuity interpretable as a shock front. For low amplitudes, these relations satisfy the Rankine-Hugoniot conditions.

Physically, a low amplitude wave of this type may be realized by setting a spherical membrane impulsively into suitable motion. For a limited range of propagation, an acoustic wave having a steep front will result.

However, to summarize the previous statements of this paper: (a) Acoustic waves cannot lead to the formation of a shock front during their propagation. (b) In contrast to high amplitude waves, acoustic waves retain their shape. (c) The wave equation (A1.1) and its solutions discussed here do not include terms which account for an energy dissipation.

The latter point deserves further elaboration. Even in the above described physical model of a very weak shock wave, energy dissipation will

become apparent when propagation over a long range is considered. The decay of the pressure will be stronger than r^{-1}, the front will broaden, and the peak will be rounded more and more, as in Figure 2.3. This means effects of viscosity will be exhibited, and we see that the acoustic treatment is an approximation which, when properly used, can be of the greatest value, but which can also lead the unwary astray.

At this time, a point may be clarified which has often caused confusion between acousticians and the workers in the field of explosions:

In Article 4.2, as well as in many other treatments of explosion shock waves, the assumption is made that the wave behind the front is sufficiently and accurately described by the equations of the non-viscous, compressible fluid motion. All viscosity effects are assumed to occur within the shock front, i.e., within the region x-x in Figure 2.3. Acousticians often claim that viscosity effects must not be restricted in this way and that they must be considered for the entire wave.

This discrepancy of opinions arises from the different ranges of propagation which are of interest in these two fields. Acousticians consider propagation over large distances where the front becomes wide and distinctly rounded and where viscosity effects are definitely noticeable behind the front. The ranges of propagation of interest in weapon effects studies are short in comparison. Here, a sharp, discontinuous shock front occurs and viscosity effects behind the front are negligibly small.

A1.5 THE BERNOULLI EQUATION AND THE KIRKWOOD-BETHE PROPAGATION THEORY. If the velocity potential, equation (A1.1), is introduced into the first hydrodynamic equation (4.1), an integration becomes possible for an isentropic flow (compare with Article 4.3):

(A1.14)
$$-\frac{\partial \varphi}{\partial t} + \frac{u^2}{2} + \int \frac{dP}{\rho} = \text{const.}$$

This is the generalized Bernoulli Equation. The integral over P resembles the Riemann Function (4.7), and it must be carried along a path of constant entropy. The magnitude is the enthalpy (heat content) of the medium designated by H or sometimes by ω. For an incompressible medium as well as the acoustic approximation, we have

(A1.15)

$$\text{Enthalpy } H = \omega = \int_{P_0}^{P} \frac{dp}{\rho} = \frac{P-P_0}{\rho} = \frac{p}{\rho}$$

$$\text{Riemann Function } \sigma = \int_{P_0}^{P} \frac{dp}{\rho c} = \frac{P-P_0}{\rho c} = \frac{p}{\rho c} \, .$$

For explosion phenomena, the medium at infinity may be considered to be at rest and having the pressure P_0. This determines the integration constant in equation (A1.14) as follows:

(A1.16)

$$- \frac{\partial \varphi}{\partial t} + \frac{u^2}{2} + \int_{P_0}^{P} \frac{dP}{\rho} = 0 .$$

If the effect of gravity is introduced and equation (A1.15) is used for the enthalpy, one obtains

(A1.17)

$$- \frac{\partial \varphi}{\partial t} + \frac{u^2}{2} + \frac{P-P_0}{\rho} + gz = 0 .$$

Here, z is the height above the level where P_0 is measured, $P_0 - \rho g z$ being the absolute hydrostatic pressure.

The equations (A1.14), (A1.16), etc., hold generally for all flow patterns if for u the total resultant velocity,

(A1.18)

$$u^2 = (\nabla \varphi)^2 ,$$

is used.

It must be stressed that the Bernoulli Equation holds only for irrotational and isentropic fluid motion.

In the acoustic approximation (A1.6) the Bernoulli Equation (A1.16) is used setting the excess pressure $p = P-P_o + g\rho z$ and omitting $u^2/2$. This omission is appropriate since the acoustic approximation considers a flow of zero Mach number $M = u/c$. Since p/ρ is of the order of magnitude uc, u^2 is of the order uM in (A1.17).

For an incompressible fluid motion, $u^2/2$ cannot be omitted in the Bernoulli Equation because it is often the most important term. Also, in calculations of bubble phenomena this term should be retained.

Kirkwood-Bethe (1942) have used the Bernoulli Equation for the development of a propagation theory for high-amplitude waves in water. The central point of this theory can be very readily stated. These workers simply set

$$(A1.19) \qquad -\frac{\partial \varphi}{\partial t} = \frac{G(\zeta)}{r}$$

and introduce this magnitude into the Bernoulli Equation. The reduced time ζ is defined by

$$(A1.20) \qquad \zeta = t - \int \frac{dr}{c+u} + const.$$

Thus G is evaluated along the (c+u)-characteristic. The calculation of ζ is not straight forward and constitutes an essential part of the theory.

Although the original Kirkwood-Bethe theory is superseded by more accurate and simpler methods, the assumption (A1.19) is still of great interest today, since it is applicable to other problems, e.g., to bubble theory. The justification of this assumption has been widely debated. First of all, it holds for isentropic processes only. Shock waves in water approximately satisfy this condition even at relatively high pressures. (For a shock wave peak pressure of around 65 kilobars, the water is $100^{\circ}C$ warmer after the passage of the front. The compressibility of water is not very

sensitive to such temperature changes. Vaporization is no problem in the domain of high amplitude waves.)

Kirkwood and Bethe have argued that equation (A1.19) and (A1.20) are exact for small disturbances as well as for incompressible fluid motion. Whitham (1953) gives the following interpretation: For the incompressible fluid motion the·characteristics degenerate into lines t = const., since c is infinite. Hence, $\partial\varphi/\partial t = - G(\zeta)/r$ is constant along the characteristics in these two cases. On this basis, the Kirkwood-Bethe approximation is a reasonable assumption. This is further supported by an error estimate which gave favorable results. However, as the authors stress, a rigorous proof that $G(\zeta)$ possesses the properties hypothetically ascribed to it appears to be prohibitively difficult.

Therefore, the Kirkwood-Bethe propagation theory is a valuable approximation and its use seems justified until it can be supplemented by a better theory. Whitham (1953) has independently obtained a theory in 1951 which has much in common with that of Kirkwood-Bethe. Both theories use the approximation (A1.19) and (A1.20).

APPENDIX B

COMMENTS ON THE ENTROPY CONCEPT

Entropy plays a profound role in almost all fields of natural sciences from biology to information theory to music. Therefore, simple explanations must necessarily fail to provide a complete picture. No wonder that some complain: "I have heard seven explanations, but did not understand a single one." With great reluctance this writer offers the following comments tailored to our specific problem.

The state of a fluid is described by two magnitudes, e.g., pressure, temperature, or density if the equation of state is given. If the specific heats are known, one can calculate the internal energy, the enthalpy, the free energy, the entropy and other variables. Any two of these magnitudes can be used equally well to describe the state of the fluid.

For our purposes, we are not interested in the entropy at a specific state, but in the entropy of a process. The second law of thermodynamics states that the entropy of a process "occurring in an isolated system" cannot decrease. It turns out that the entropy change is a quantitative measure of the irreversibility of the process.

To elaborate on the concept of the isolated system, consider four examples, (a) through (d).

(a) Visualize a completely gas-tight piston which moves in a gas-filled cylinder. All inside walls are equipped with a perfect thermal insulation. If we push the piston inward* the pressure and temperature of the gas will rise. Owing to the perfect insulation, there will be no heat conduction to the walls. Upon returning to the original position of the piston, the

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* The motion must be slow so that the formation of shocks is prevented.

initial pressure and temperature are obtained: the process is reversible and the term "isolated system" refers in this case to the insulation which prevented a heat loss by conduction.

(b) Assume that the walls of the cylinder are not insulated and that they are cooled. As the temperature of the gas rises upon compression, heat will be transferred to the cooled walls. If the piston is returned to the position where the original pressure is reached, the temperature will be below its initial value. This process is irreversible. The temperature difference becomes larger and larger as the number of compressions is increased. The entropy change of such a process is

$$(A2.1) \qquad \Delta S = c_p \ln \frac{T_o}{T_1}$$

where c_p = heat capacity at constant pressure, T_o = initial temperature, and T_1 = temperature at the end of the cycle.

In this case, the isolated system is comprised of the cylinder and the cooling agent.

(c) We return to the insulated cylinder and assume that it is equipped with a narrow nozzle through which the gas is pressed. Such a nozzle will produce a drag, i.e., a dissipation of the mechanical energy. Consequently, the temperature is higher when the piston is returned to the point of the initial pressure. This process is also irreversible. The entropy change in this case is

$$(A2.2) \qquad \Delta S = c_p \ln \frac{T_1}{T_o}$$

(in thermodynamics delicate rules determine the signs of the variables, so that the signs of ΔT are opposite in (A2.1) and (A2.2)).

Mechanical work must be done to move the piston. In the reversible case the total work of compression and expansion is zero. In the other two

cases, a finite amount of work must be expended to bring the cylinder back to the position of the initial pressure. In one case the energy difference is found in the heated cooling agent, in the other case, in the heated gas. In practical machines this energy is usually a total loss. The second law of thermodynamics states that this energy is degraded and has lost a portion of its ability to do mechanical work. One may think of vaporizing the cooling water and using it to drive a steam engine. But only a fraction of the energy transmitted to the cooling water can be recovered by such a device. The entropy is a measure of this energy degradation. No degradation occurs in a reversible process, but almost all processes in nature show a degradation.

The opposite of an irreversible process (which amounts to an upgrading) is a physical impossibility called a perpetuum mobile of the second kind. In this case, the entropy would decrease.

The term "adiabatic" refers to processes in insulated systems, i.e., absence of heat conduction. Example (c) shows that an adiabatic process must not necessarily be an isentropic one. It is interesting to note that this very obvious distinction between adiabats and isentropes came only very recently (around World War II) into common usage.

When proceeding to Part III of this paper, the reader will notice that process (a) corresponds to common sound waves (Laplace), process (c) to shock waves, and that process (b) is related to Newton's approach of describing pressure waves.

(d) Although not essential for our purposes, we shall complete the discussion of the concept of the "isolated system" by considering an internal combustion engine. Assume that heat is added to the gas (e.g., by combustion) when the piston has reached the innermost position. For simplicity, we assume that the cylinder is isolated. (The net mechanical work done by the piston is positive, i.e., this device transforms thermal into mechanical energy.) In this case, one finds that the entropy of the gas has increased. This is because the isolated system comprises not only the

gas in the cylinder, but also the heat source. The entropy of the combined system increases.

If the heat is to be provided by the combustion of gasoline, the question of the entropy of gasoline will arise. It is not very different from that of water. In our case, the latent chemical energy of fuel must be considered. Carrying out the pertinent calculations, one will find a substantial degradation of the ability of gasoline to do work.

In this sense, one must understand the statement that the entropy of the universe increases and approaches a maximum: First, the universe is clearly an isolated system and secondly, almost all natural processes are irreversible, i.e., there is a continuous degradation in the sense stated above.

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13. ABSTRACT

ABSTRACT: This paper gives a narrative description of the hydrodynamic concepts which are important for the understanding of underwater explosion processes with particular emphasis given to the physical background. Mathematical developments are entirely omitted or kept to a minimum.

The paper describes the concepts of the various fluid motions and fluid models involved in explosion phenomena. The properties of the shock front are described and the interrelationship between the formation of the nuclear bubble and the shock front is pointed out. The properties of high amplitude waves are outlined using Riemann's description. The formation of underwater explosion bubbles is shown to be a hydrodynamic consequence of the spherical pressure wave emitted by the explosion. Such a wave always produces a radial mass flow directed outward, the afterflow, which must lead to the formation of a cavity. The acoustic approximation of pressure waves is discussed in Appendix A. Appendix B contains comments on the entropy concept.

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